

1997

# Strategic bidding in an energy brokerage

Sundar Rajan  
*Iowa State University*

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>



Part of the [Electrical and Computer Engineering Commons](#), [Finance Commons](#), and the [Finance and Financial Management Commons](#)

---

## Recommended Citation

Rajan, Sundar, "Strategic bidding in an energy brokerage " (1997). *Retrospective Theses and Dissertations*. 12027.  
<https://lib.dr.iastate.edu/rtd/12027>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact [digirep@iastate.edu](mailto:digirep@iastate.edu).

## **INFORMATION TO USERS**

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

**The quality of this reproduction is dependent upon the quality of the copy submitted.** Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

# **UMI**

A Bell & Howell Information Company  
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA  
313/761-4700 800/521-0600



Strategic bidding in an energy brokerage

by

Sundar Rajan

A dissertation submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
DOCTOR OF PHILOSOPHY

Major: Electrical Engineering (Electric Power)

Major Professor: Dr. John Lamont

Iowa State University

Ames, Iowa

1997

Copyright © Sundar Rajan, 1997. All rights reserved.

**UMI Number: 9814688**

---

**UMI Microform 9814688**  
**Copyright 1998, by UMI Company. All rights reserved.**

**This microform edition is protected against unauthorized  
copying under Title 17, United States Code.**

---

**UMI**  
**300 North Zeeb Road**  
**Ann Arbor, MI 48103**

Graduate College  
Iowa State University

This is to certify that the Doctoral dissertation of  
Sundar Rajan  
has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

**Major Professor**

Signature was redacted for privacy.

**~~For the~~ Major Program**

Signature was redacted for privacy.

**~~For the~~ Graduate College**

## TABLE OF CONTENTS

<b>1</b>	<b>INTRODUCTION . . . . .</b>	<b>1</b>
1.1	Problem Overview . . . . .	2
1.1.1	Statement of Research Objectives . . . . .	2
1.1.2	Brokerage Rules and Assumptions . . . . .	4
1.1.3	Factors Affecting Bidding . . . . .	7
1.1.4	Strategic Bidding . . . . .	11
1.2	Executive Summary . . . . .	11
1.3	Overview of Original Contributions . . . . .	13
<b>2</b>	<b>REVIEW OF RELEVANT LITERATURE . . . . .</b>	<b>16</b>
2.1	Power Industry Restructuring and its Impacts . . . . .	16
2.2	Energy Markets – Implementations and Rules . . . . .	17
2.3	Strategic Bidding – Other Industries . . . . .	22
2.4	Strategic Bidding – Power Industry . . . . .	24
2.5	Generation and Transactions Scheduling . . . . .	27
2.6	Transmission Access and Pricing . . . . .	28
2.7	New Tools and Concepts in Energy Markets . . . . .	30
<b>3</b>	<b>MARKET-BASED STRATEGIES . . . . .</b>	<b>32</b>
3.1	Perfect Competition . . . . .	32
3.2	Imperfect Market Conditions in Energy Markets . . . . .	33
3.3	Modeling Competitor Behavior . . . . .	34

3.3.1	Using Bid Distributions – An Illustrative Example . . . . .	35
3.3.2	Development of a Suboptimal Bidding Strategy . . . . .	36
3.3.3	A Numerical Example . . . . .	40
3.4	Polynomial Modeling of a Bid Distribution . . . . .	42
3.5	Incomplete-Beta Function Modeling of a Bid Distribution . . . . .	46
3.6	Suboptimal Bidding Using the Incomplete-Beta Function . . . . .	49
3.7	Incomplete-Gamma Function Modeling of a Bid Distribution . . . . .	52
3.8	Suboptimal Bidding Using the Incomplete-Gamma Function . . . . .	54
3.9	Other Candidates for Modeling a Bid Distribution . . . . .	56
3.9.1	The Pareto Distribution . . . . .	57
3.9.2	The Exponential Distribution . . . . .	58
3.9.3	The Weibull Distribution . . . . .	58
<b>4</b>	<b>BROKERAGE SIMULATOR . . . . .</b>	<b>60</b>
4.1	Rationale for Simulation . . . . .	60
4.2	Simulator Overview . . . . .	62
4.2.1	Bid Matching Module . . . . .	64
4.2.2	Transmission Network Module . . . . .	66
4.2.3	Bid Builder Module . . . . .	71
4.2.4	Other Bid Sources . . . . .	71
4.2.5	Bid Development Module . . . . .	71
4.2.6	Profit Calculation Module . . . . .	73
4.2.7	Regression Analysis . . . . .	74
4.2.8	Heuristics . . . . .	75
<b>5</b>	<b>SIMULATION RESULTS AND ANALYSIS . . . . .</b>	<b>76</b>
5.1	Simulator Use Overview . . . . .	76
5.2	Overview of the Test System . . . . .	78



5.3	Initial Calculations and Simulations . . . . .	80
5.4	Polynomial Model . . . . .	83
5.5	Incomplete-Beta Model . . . . .	89
5.6	Variations on Competitor Strategies . . . . .	98
5.7	Effects of the Transmission Network Module . . . . .	101
5.7.1	Light Line Loading Conditions . . . . .	101
5.7.2	Heavy Line Loading Conditions . . . . .	104
<b>6</b>	<b>SCHEDULING-BASED STRATEGIES . . . . .</b>	<b>105</b>
6.1	Scheduling Considerations in Bidding . . . . .	105
6.2	Factors To Be Considered in Scheduling Decisions . . . . .	108
6.3	Effects of Schedule Changes on Economics and Operations . . . . .	109
6.4	Modeling Risk Preference in Scheduling Decisions . . . . .	109
6.4.1	Classification of Risk . . . . .	110
6.4.2	Utility Functions . . . . .	111
6.4.3	Modeling Risk in Strategic Bidding Using Utility Functions . . . . .	115
6.5	Scheduling Considerations – Illustrative Scenarios . . . . .	118
6.5.1	Change in Generation Requirements . . . . .	118
6.5.2	An Increase in Reliability Requirements . . . . .	120
6.5.3	A Change in Maintenance Requirements . . . . .	121
6.5.4	Unit Environmental Requirements . . . . .	122
6.5.5	Unit Fuel Considerations . . . . .	123
6.5.6	Efficiency Considerations . . . . .	124
6.5.7	Market Condition Considerations . . . . .	125
6.5.8	Secondary-Effect Considerations . . . . .	126
6.6	Scheduling Considerations – Numerical Examples . . . . .	126
6.6.1	Selling/Buying Without Changing Commitment Order . . . . .	126

6.6.2	Avoiding Startups/Shutdowns . . . . .	129
6.6.3	Simultaneous Buy/Sell Activity . . . . .	138
6.6.4	Consideration of Take-or-Pay Fuel Contract Requirements . . . .	141
6.6.5	System Health Considerations . . . . .	142
<b>7</b>	<b>SUGGESTIONS FOR FUTURE WORK AND CONCLUSIONS .</b>	<b>146</b>
7.1	Suggestions for Future Work . . . . .	146
7.1.1	Market-Based Strategies . . . . .	146
7.1.2	Scheduling-Based Strategies . . . . .	148
7.2	Conclusions . . . . .	149
7.2.1	Market-Based Strategies . . . . .	149
7.2.2	Scheduling-Based Strategies . . . . .	152
<b>APPENDIX</b>	<b>. . . . .</b>	<b>154</b>
<b>BIBLIOGRAPHY</b>	<b>. . . . .</b>	<b>159</b>

## LIST OF TABLES

Table 1.1	Factors affecting the bidding strategies of the participants. . . .	8
Table 3.1	Variation of suboptimal values with marginal cost – triangular distribution. . . . .	41
Table 3.2	Variation of suboptimal values with marginal cost – polynomial distribution. . . . .	45
Table 3.3	Variation of suboptimal values with marginal cost – beta distribution. . . . .	51
Table 3.4	Variation of suboptimal values with marginal cost – gamma distribution. . . . .	56
Table 5.1	Test system data – company summary information. . . . .	79
Table 5.2	Polynomial model – all-participant profits in \$. . . . .	83
Table 5.3	Polynomial model – suboptimal bidding profits in \$. . . . .	85
Table 5.4	Modified polynomial model – all-participant profits in \$. . . . .	87
Table 5.5	Modified Polynomial model – suboptimal bidding profits in \$. . .	88
Table 5.6	Incomplete-beta weekly model – all-participant profits in \$. . . .	90
Table 5.7	Incomplete-beta weekly model – suboptimal bidding profits in \$. .	91
Table 5.8	Incomplete-beta daily model – all-participant profits in \$. . . .	92
Table 5.9	Incomplete-beta daily models – suboptimal bidding profits in \$. .	92
Table 5.10	Incomplete-beta average model – all-participant profits in \$. . . .	93
Table 5.11	Incomplete-beta spread model – all-participant profits in \$. . . .	97

Table 5.12	Simultaneous strategy simulations – all-participant profits in \$.	99
Table 5.13	Light line loading – all-participant profits in \$.	102
Table 5.14	Transmission modeling – suboptimal bidding profits in \$.	102
Table 5.15	Heavy line loading – all-participant profits in \$.	104
Table 6.1	Reasons for changing commitment schedule.	106
Table 6.2	Effects of changing commitment schedule on economics.	109
Table 6.3	Effects of changing commitment schedule on operations.	109
Table 6.4	Variation of suboptimal values of 4-unit system with strategy.	133
Table 6.5	Variation of suboptimal values of 16-unit system with strategy (purchase case).	135
Table 6.6	Variation of suboptimal values of 16-unit system with strategy (sale case).	138
Table 6.7	Variation of suboptimal values for simultaneous buy/sell – Case 1.	140
Table 6.8	Variation of suboptimal values for simultaneous buy/sell – Case 2.	140
Table 6.9	Variation of suboptimal values for simultaneous buy/sell – Case 3.	141
Table 6.10	Variation of suboptimal values for take-or-pay fuel block consid- eration.	142
Table 6.11	Variation of suboptimal values of 16-unit system with strategy (sale case considering unit outage possibility).	145

## LIST OF FIGURES

Figure 3.1	Triangular PDF for buy bids. . . . .	37
Figure 3.2	Triangular PDF for sell bids. . . . .	40
Figure 3.3	Fitting a polynomial to bid data. . . . .	43
Figure 3.4	Incomplete-beta function. . . . .	48
Figure 3.5	PDF from incomplete-beta CDF. . . . .	48
Figure 3.6	Variation of $E_{lb}(x)$ with $x$ . . . . .	51
Figure 3.7	Incomplete-gamma function. . . . .	53
Figure 3.8	PDF from incomplete-gamma CDF. . . . .	54
Figure 3.9	Variation of $E_{lb}(x)$ with $x$ . . . . .	55
Figure 4.1	Brokerage simulator – functional overview. . . . .	63
Figure 5.1	Modified IEEE-RTS system . . . . .	81
Figure 5.2	Sensitivity of suboptimal solution to $m$ . . . . .	95
Figure 5.3	Sensitivity of suboptimal solution to $a$ . . . . .	95
Figure 5.4	Sensitivity of suboptimal solution to $b$ . . . . .	96
Figure 6.1	Expo-Power utility: variation with $\alpha$ . . . . .	116
Figure 6.2	Expo-Power utility: variation with $\beta$ . . . . .	116
Figure 6.3	Variation of coefficient of absolute risk aversion with $\alpha$ . . . . .	117
Figure 6.4	Variation of optimal solution with wealth: CARA. . . . .	128
Figure 6.5	Variation of optimal solution with wealth: IARA. . . . .	128

Figure 6.6	Variation of optimal solution with wealth: DARA. . . . .	129
Figure 6.7	Base case unit commitment for 4-unit system. . . . .	131
Figure 6.8	Purchase case unit commitment for 4-unit system. . . . .	131
Figure 6.9	Base case unit commitment for 16-unit system. . . . .	134
Figure 6.10	Purchase case unit commitment for 16-unit system. . . . .	134
Figure 6.11	Sale case unit commitment for 16-unit system. . . . .	137
Figure 6.12	Simultaneous buy/sell: three possible cases. . . . .	139
Figure 6.13	Base case unit commitment for 16-unit system (with unit outage). 143	
Figure 6.14	Sale case unit commitment for 16-unit system (with unit outage). 143	

## LIST OF SYMBOLS

$a_0, a_1, \dots, a_{n+1}$	Coefficients of the polynomial CDF
$b_{min}, b_{max}$	Lower and upper bounds for triangular PDF domain
$B(a, b)$	Beta function with parameters $a$ and $b$
$c$	Marginal cost of production of a block of energy (\$/MWH)
$\hat{c}$	Marginal cost of production of a block of energy after unit outage (\$/MWh)
$C$	Generation production cost function
$E_{lb}(x)$	Lower bound to expected profit from a normalized bid $x$ (\$)
$f(X)$	PDF of random variable $X$
$F(X)$	CDF of random variable $X$
$I_x(a, b)$	Incomplete-Beta function with parameters $a$ and $b$
$m$	Normalizing bid price for incomplete-beta model (\$/MWH)
$n$	Degree of polynomial used for polynomial model
$p_b, p_s$	Buy and sell bid prices (\$/MWH)
$p_b^*, p_s^*$	Suboptimal buy and sell bid prices (\$/MWH)
$p_{f.o.r.}$	Probability of forced outage of a unit
$P(a, x)$	Incomplete-Gamma function with parameter $a$
$q$	Quantity or block of energy bought or sold (MWH)
$r$	Arrow-Pratt index of absolute risk aversion
$S(x)$	Probability of acceptance of a bid expressed as a function of normalized bid price $x$

$U(w)$	Utility function expressed as function of wealth $w$
$w$	Wealth (profit) resulting from a bid (\$)
$W$	Initial wealth prior to evaluation of the bid (\$)
$x$	Normalized bid price for incomplete-beta model
$x^*$	Suboptimal normalized bid price for incomplete-beta model
$X$	Random variable representing competitors' bids
$\alpha, \beta, \theta$	Parameters of Expo-power utility function
$\Gamma$	Gamma function with parameter $a$
$\pi$	Profit from a transaction



## ACKNOWLEDGEMENTS

I would like to express my gratitude to my parents and to my sister for their support throughout my life and especially during the hard times. My sincere thanks to my major professor, Dr. John Lamont, for his guidance and for having confidence in me. Thanks are also due to the members of my committee for their suggestions and objections, that helped improve my thought process, and hopefully, the quality of this research.

I would like to thank and acknowledge my fellow students Sridhar Kondragunta, Bradley Nickell, and Fu Jian for their support in developing some of the software modules used in this research.

Special thanks to Srinu Battula and Manos Obessis for their loyal friendship, moral support, and timely humour that made graduate school less of a chore and the Ph.D. program almost tolerable.

Most of all, I would like to thank my wife Girija, from the bottom of my heart, for being there during all the times that she was needed. Without her support and patience, this work would not have been possible.

## 1 INTRODUCTION

With the proposed restructuring of the electric energy markets in the United States and in other countries of the world, several new market structures are emerging for the buying and the selling of electric energy. One such market structure is the energy brokerage, wherein buyers and sellers of electric energy submit bids to buy and sell energy to a central entity known as the broker. The broker then proceeds to match buyers and sellers to form binding transactions, subject to system constraints, such that the total savings resulting from the matching process is maximized. The resultant savings are then allocated among the various participants in a predetermined, and presumably, equitable manner.

The subjects of how to perform the matching, and how to allocate the resultant savings are already well-researched areas in the literature. These will not be the focus of this research. The subject of how participants in the brokerage might use the available information in a strategic manner, and submit the resulting bids to the broker that differ from their marginal costs, is known as “strategic bidding”, and is a relatively new topic in the energy market context. The reason for this could be the fact that, in the past, utilities, and to a lesser extent, independent power producers, were assured of a net profit virtually irrespective of their costs. This was the underpinning of the regulated monopolies that were in existence until the so-called deregulation movement started. One of the effects of such a system was to reduce electric energy pricing to “merely” a production costing operation<sup>1</sup>.

---

<sup>1</sup>It is, of course, well known that even this production costing function is complex, uncertain and

This subject of strategic bidding was the focus of the research reported in this dissertation. Thus, the work presented here primarily deals with developing methods that *individual* participants could use in bidding into an energy brokerage market. Preliminary bidding strategy development as a part of this research was reported in [1] and [2].

## 1.1 Problem Overview

In this section, an overview of the work performed in this research is presented. This overview includes a statement of research objectives, the brokerage rules and assumptions used in the market modeling, an examination of the factors that affect bidding and the definition of scope for this research, and a brief description of the bidding strategies developed in this research.

### 1.1.1 Statement of Research Objectives

The main contribution of this research is the definition, and the demonstration of use, of a framework for the development and evaluation of bidding strategies, for participants to use, in preparing and submitting bids to an energy brokerage market.

The framework includes:

1. The rules under which the market operates.
2. The different types of participants.
3. The different objectives of these participants.
4. The factors that affect the bidding of the participants.
5. Strategies that consider the factors in item 4 and achieve these objectives.

---

often inaccurate. However, the deregulated future promises to make the overall pricing process an even more complex and academically interesting activity.

6. A tool to simulate market conditions, including competition from other participants, with which to test these strategies.

The various participants in the energy brokerage market could use this framework to evaluate and improve their bidding strategies and, thus, their economic performance according to their objectives.

Of the above items, item 1, while not clearly defined in all aspects, is already a fairly well-researched topic in the literature. So, a subset of the rules reviewed from existing literature has been assumed in this research. Items 2 and 3 are less complex and are well-defined in the literature. Examples of the types of participants defined in the literature include investor-owned utilities, independent power producers, generation companies, distribution companies, large industrial customers, power marketers and load aggregators. These types could also be classified according to their functions as well as according to their performance goals. Some aspects of items 4 and 5 have been approached by other researchers in the power industry, and in other industries. This research aims for contributions that enhance these approaches, or adapt them for application to the power industry. Examples include the use of historical market information in bidding, and the modeling of competitors' behavior. Some aspects of items 4, 5, and 6 are relatively new topics in the energy brokerage context. Examples include the effects of generating unit availability, startup/shutdown considerations, load forecast errors, and transmission considerations. Of these, the effects of unit availability and startup/shutdown considerations have been analyzed in 6. Admittedly, the modeling presented for these analyses is very simple. But as will be illustrated in that chapter, even for the simple model, the implications of including these considerations can be very complex.

In addition, the tools developed for brokerage simulation and strategy evaluation are expected to be a useful contribution for future research.

Thus, the primary focus of this research is to develop strategies for various types

of *individual participants* in the energy brokerage to use in order to bid successfully. This research does not propose new models for *implementation* of energy brokerages. Rather, assumptions are made regarding brokerage implementation based on models already developed by other researchers. Given such a brokerage market, the strategies developed will aim to improve individual participants' goals within the rules of the market. To test these strategies and to draw insights from simulations, we needed a tool that simulates a brokerage. One such tool is the brokerage simulator developed as part of this research. Again, while the simulator is expected to be helpful in testing of the strategies, the strategies will be independent of the simulator, and will be applicable to participants who choose a different (presumably more advanced) tool for evaluation. The contribution of this research includes original ways to utilize the information generated by the simulator.

### 1.1.2 Brokerage Rules and Assumptions

This section summarizes the rules and assumptions that are used in modeling the brokerage market. The rationale for these assumptions is explained in Chapter 2, in the context of a review of the relevant literature.

1. The brokerage market is assumed to be a facilitator for exchange of bulk energy (real power and energy only) between participants. In general, these participants could be buyers or sellers of energy. The primary focus of the strategy development in this research is for participants who have generating resources, and either have a means to calculate their production costs, or have a way to determine the replacement cost/value of energy from a source outside the market. A participant who is a pure load can still use these same strategies. However, load-side issues such as demand side management and direct load control are not explored in this research.

2. Participants submit bids to buy or sell predetermined blocks of hourly energy. The bids include the participant name, the hour for the proposed purchase or sale, the number of blocks, whether the bid is for a sale or a purchase, and the bid price.
3. The broker accepts the bids and matches them according to the Florida high-low matching method. The bidding is considered to be one-shot; in other words, multiple rounds of bidding are not performed. This method is explained Chapter 2. The transaction price is set to be the average of the buy and sell bid prices.
4. The matching stops when the buy bid price no longer exceeds the sell bid price, i.e., when no additional savings can be achieved.
5. In the cases where the transmission network is modeled, it is assumed that the broker performs a DC-power flow-based calculation for all the transactions that result in potential savings. Thermal line limit constraints are enforced. Transactions that violate these limits are rejected.
6. Transmission usage costs are calculated by using the MW-mile method, which is a distance-based incremental flow method. These costs are split equally between the two parties of a successful match. If the potential savings from the energy price differential between the bids is less than the potential transmission usage costs, the transaction is rejected.
7. The resultant match information is distributed only to the two parties involved in the transactions, i.e., bidding is sealed, and the bid information is assumed to be private and protected.
8. The broker reports the resulting transaction prices, and if applicable, the transmission usage costs, from each transaction, in a public database that is equally accessible by all participants. Also reported in this database, are the line flows in each line at the end of each hour.

The above assumptions are a summary of the key assumptions made in this research. Other assumptions made primarily involve the detailed modeling of the market and the strategies, and will be stated and clarified in the appropriate sections. Of the above assumptions, two issues merit further comment. These are:

- *Ancillary Services:*

Exactly what the term *ancillary services* means is not sharply defined at the time of writing this dissertation. However, the Federal Energy Regulatory Commission (FERC) identified through its order no. 888, at least six services that a transmission provider must include in an open access transmission tariff. These are, (1) energy imbalance service, (2) spinning reserve, (3) supplemental reserve, (4) reactive supply and voltage control, (5) regulation and frequency response, and (6) scheduling, system control and dispatching services. In addition, four other services were also identified, that the transmission provider may offer as optional services. These are (1) backup supply service, (2) dynamic scheduling service, (3) real power loss service, and (4) restoration service. These services are not considered as part of the brokerage market in this research.

- *Spinning Reserves:*

The issue of spinning reserves has been indirectly approached in this research as follows. It was assumed that each participant in the test system was required by an entity outside the brokerage to carry 15% of the hourly forecast load as spinning reserves, and this was enforced as a constraint during the unit commitment phase of the individual companies. However, the issue of spinning reserves to support the transactions occurring as a result of successful matches has not been included. The effect of spinning reserve requirements from new transactions will have an effect on the strategies of participants, and mechanisms to include this effect is a candidate for future research.

### 1.1.3 Factors Affecting Bidding

In this section, the various factors that could affect the bidding behavior of the various participants are examined. From this examination, the scope is defined for the research described in this dissertation.

The following parts of this section explain the impact of each of the factors on bidding strategies.

#### 1.1.3.1 Market-Related Factors

*Distribution of prices:* Estimated price distributions serve the role of a forecast of market prices in the following bidding periods, and will impact the optimal bidding strategies of the players. One example of an impact is for the case of a seller who deterministically knows the cost of his generation and is trying to determine the optimal markup for bidding. This factor is well researched in other industries, and can be easily modified for power industry application.

*Expected savings/profits:* Participants that wish to model competition might use the above distribution of prices to estimate probability of acceptance of their bids. This estimate is then used to derive an expected value of profit, which is then maximized.

*Company conditions compared to market conditions:* This factor determines if the participants have any opportunity to wield market power and influence their profits. For example, if the market is short on supply, and the participant is long on supply, then the participant might be able to take advantage of the buyers by bidding high.



Table 1.1 Factors affecting the bidding strategies of the participants.

<b>FACTOR</b>	<b>SELLER</b>	<b>BUYER</b>
<b>MARKET-RELATED</b>		
Distribution of prices	WS <sup>a</sup>	WS
Expected savings/profits	WS	WS
Company conditions compared to market conditions (Energy shortage/surplus/neutral)	WS	WS
<b>SCHEDULING-RELATED</b>		
<i>Generating unit related:</i>		
Online generation reserves	WS	WS
Unit startup/shutdown	WS	WS
Unit availability	WS	WS
Unit maintenance requirements	OS <sup>b</sup>	OS
<i>Load-related:</i>		
Load forecast error risk	OS	OS
Load curtailment/DSM/DLC	OS	OS
Weather and other external factors	OS	OS
<i>Transmission network related:</i>		
System network condition	OS	OS
Company network condition	OS	OS
Transmission usage costs	OS	OS
Transmission losses costs	OS	OS
Network outage risk	OS	OS
Ancillary services usage and costs	OS	OS
<i>Fuel-related:</i>		
Fuel supply/network availability	OS	OS
Fuel price risk	OS	OS
Fuel contract limits	OS	OS
Take-or-pay constraints	OS	OS
On-hand fuel supply	OS	OS
<i>Emission-related:</i>		
Compliance costs	OS	OS
Emissions allowance availability	OS	OS
Emission allowance value	OS	OS
Emission production	OS	OS

<sup>a</sup>WS = Within Scope<sup>b</sup>OS = Outside Scope

### 1.1.3.2 Scheduling-Related Factors

*Generating unit related factors:* Unit availabilities, including maintenance requirements, affect both bidding behavior and eventual profit from bidding by introducing uncertainty in participants' supply/demand conditions. If participants consider these uncertainties, then their bidding strategies will be affected by this risk. Generation reserves or online excess capacity is a factor that affects the marginal costs of the participants, and hence will affect bidding strategies. Traditionally, generation scheduling is performed to minimize the operating costs with respect to constraints. However, if the objective function is now to maximize (expected) profits, then startup and shutdown cost considerations will affect strategies if the participants wish to evaluate opportunities to take advantage of market prices to a greater extent, and perform generation scheduling that is focused toward the marketplace. There is also the potential impact of minimum up/down times of the units started/shutdown on marginal costs in future bidding hours. Again, this process involves risk, and it is proposed in this framework to identify how this risk can be quantified.

*Load-related factors:* Load forecast errors, including weather and other external factor effects, will have a similar effect as unit availabilities in that they introduce uncertainties in supply/demand conditions. Thus they also affect the risk involved with bidding strategies. Load curtailment agreements, demand side management (DSM), and direct load control (DLC) all have complex effects on the uncertainties involved with demand, and thus will impact bidding strategies. However, they are outside the scope of this research.

*Transmission network related factors:* System network conditions are expected to be public domain information. These will affect the participants' decisions by affecting the

feasibility of transactions. Transmission usage costs, losses costs, and ancillary service costs will affect participants' bid outcomes because they affect the overall economics of the bid matching process. While there is very little information on how small or large this impact will be in the future, an attempt was made in this research to incorporate their effects on the effectiveness of the bidding strategies of the participants. Network outage risk is a major aspect that affects system operation, and will have an impact on participants' bidding strategies. However, it is outside the scope of this research.

*Fuel-related factors:* Internal and external fuel supply as well as fuel network capacity and availability obviously affect the production costs, and thus affect the participants' bidding strategies in an indirect way. The volatility of fuel prices might introduce errors in the participants' estimates of production costs, and might have an impact on strategic bidding. However, many participants consider fuel price risk separately from the scheduling function. For example, the participants may hedge fuel prices by purchasing fuel options. Thus, as far as bidding in the energy brokerage market they might consider fuel prices to be deterministic. However, this might change in the future. For now, fuel considerations are outside the scope of this research, while being a candidate for a future research topic.

*Emission-related factors:* The impact of transactions on emissions will affect compliance strategies and costs, and therefore the overall profitability of the transaction. Also, if the emission allowance (EA) market trading becomes significant in the future, then the value of the EA will affect costs. For now, emission considerations are also outside the scope of this research.

Some of the above factors may not have a direct impact on certain types of players. For example, if the player owns no generation and is a pure buyer, or a power marketer,

then none of the generating unit related aspects affect him.

#### **1.1.4 Strategic Bidding**

As outlined in Chapter 2, different approaches have been suggested in the literature as possible methods for the participants to employ in an auction market such as the energy brokerage. The approaches presented in this research essentially involve modeling competitors' bidding behaviors by probability distributions, followed by maximizing a lower bound to expected profits from the bidding activity. Thus, the implicit assumption is that participants seek to maximize not only their profit from the bidding activity, but also wish to achieve an optimal probability of acceptance. Thus, the objective function of the participants is assumed to be the expected profit, which is the product of the profit, and the probability of achieving this profit, given that the competitors bid according to an estimated probability distribution.

In Chapter 6, some illustrative scenarios are presented, that extend this expected profit maximization approach to an expected utility maximizing approach. This provides a method to include the concept of risk preference in the bidding strategies.

## **1.2 Executive Summary**

In Chapter 2, a review of the relevant literature is presented. Because of the wide scope of the strategic bidding problem, the volume of literature that could arguably be "relevant" to the research is very large. Thus, that chapter is not intended to be an comprehensive bibliography on the issues that affect strategic bidding. However, an attempt has been made to provide a review of a representative sample of the papers from each relevant area.

In Chapter 3, strategies that attempt to include competitor behavior by using available market information are presented. A lower bound on the profit from bidding is

derived, which is useful in providing an objective function that can be optimized using the limited information assumed to be available in this research. This is followed by derivations for optimal bids that maximize this lower bound, for different assumptions about the probability distribution of the competitors. Details on the optimality and concavity conditions are presented in the Appendix.

In Chapter 4, the brokerage simulator, which was developed as a part of this research, is described. This simulator is an implementation of the rules assumed in this research. It was used to test the strategies developed and presented in Chapter 3. Results of the simulations performed on this simulator are presented and analyzed in Chapter 5. Also, based on these results, some heuristics were developed to improve the performance of the strategies. Results from implementing these heuristics are also presented in this chapter.

In Chapter 6, a qualitative treatment of the scheduling factors that might affect bidding strategies is presented, followed by numerical examples to illustrate these effects. Also included in this chapter, is a treatment of risk preferences by using results from recent developments in the field of utility theory and risk preference functions by researchers in economics. This is followed by the modeling of bidding objectives as expected utility maximizations, and the comparison of results from using this type of objective, to using the expected profit maximization objective, for various scheduling scenarios. These scenarios are to be viewed as a first attempt at modeling scheduling risk considerations in strategic bidding, and not as solutions prescribed to handle the rather complex considerations of risk in the energy markets.

Conclusions and suggestions for future work are presented in Chapter 7.

### 1.3 Overview of Original Contributions

This section provides an overview of the original contributions made by this research to the power systems area. While periodically in this dissertation, we make an attempt to clarify which parts of the work presented are results of assumptions and lessons drawn from existing literature, and which parts are results of original work by this author, this section clearly states the primary original contributions.

1. The primary contribution of this research is the development of a framework for developing bidding strategies for individual participants. This framework includes suggested ways to model competitors, initial development, implementation and testing of strategies, and the development of a tool to simulate such strategies. Given that bidding strategies in energy markets is a hitherto less researched area, it is expected that several researchers are developing different frameworks to achieve these tasks. Therefore, no claims are made by this author as to the superiority of the approaches presented herein; only the *originality* of the contributions to power system research is presented here.
2. The modeling of competing bids as a probability distribution is an idea that has been used in several other industries. However, with the exception of [3], the work presented here is one of the first attempts, to the best of this author's knowledge, to apply this technique to energy brokerages. Reference [3], while being published at a time when this research was in its early stages, is an independent work, and did not serve as a model for our work.
3. The consideration of buyer-side bidding is omitted in [3]. Buyer-side bidding is included in our considerations. As a way to avert the computational complexities in considering two different kinds of distributions at the same time, our contribution is the lower bounding of the profit, followed by the application of heuristics to

fine-tune the bid. This is also a new development in energy brokerage bidding strategies.

4. The detailed derivation of the first- and second-order conditions required for the implementation of the suboptimal bidding procedure, for the polynomial, the beta and the gamma distribution cases, were performed by this author.
5. Several different simulators that use various models to implement energy brokerages have already been implemented by other researchers, and it is fair to assume that many more are under development. However, in this dissertation, we have developed original heuristics that adjust the bid prices in response to the base case results. Both the heuristics themselves, and the general framework of the simulator that helps to develop these heuristics easily, are original contributions of this author, and are expected to be of value to the power industry.
6. The use of utility functions to model risk in transactions is a well-researched area in economics. However, the use of the results from work by Saha [4], in the form of the flexible Expo-power utility function, to model the effects of risk preference in energy brokerage transactions is original to this work. While the extent of modeling presented herein is limited, such an approach can be explored by researchers in a variety of power system risk researchers, and could be of value in several areas in addition to power system operations. It must be mentioned here that in a related area of transaction selection, Kumar et al [5] presented a framework for transaction selection using decision analysis. However, the utility function used there was purely exponential in nature, and is not flexible enough to model various risk preference attitudes. In our work, we chose the Expo-power function because of its flexibility. This flexibility is needed in order to allow enough latitude in our framework for participants of widely different risk preference attitudes, since

very little is known currently about the kind of participants that will evolve as the primary players in energy markets.

Once again, it is emphasized that the purpose of this section is only to specify the original contributions made by this research. The other research projects referred to in the above item are expected to be ongoing projects, and the comparisons made in those items are based on published work only. The comparisons serve the purpose of distinguishing our work from the work presented in those references, and are not made with a view of criticizing those references in any manner.

Also, although progress has been made in the areas described in the above items, none of the problems described in this research is considered to be solved completely. Future research will probably expand the results presented here. However, significant contributions have resulted from this research.



## 2 REVIEW OF RELEVANT LITERATURE

### 2.1 Power Industry Restructuring and its Impacts

In this section the papers reviewed cover the impact of the market restructuring on various aspects of the power industry. Kriz [6] provided an overview of the advent of competition in several states including California, Massachusetts, Rhode Island, and New Hampshire. In that paper, the author outlined some of the legislative complexities involved in the process of deregulation, such as the issue of jurisdiction of various government bodies like FERC, Congress, state and local governments. The paper also provided a table of the electricity rates prevalent in 20 states with the highest and lowest 10 rates being shown. It is not surprising to note that of these states, some with the highest rates (New Hampshire, Rhode Island, Massachusetts and California) also have the hottest debates going on, regarding restructuring of the local energy markets.

Dunn et al [7] broadly identified, the impacts of restructuring the energy markets on the objectives of the participants, control center applications, information technology requirements, and transactions analysis. In that paper, the authors identified some key issues such as the value of information, the possibility of gaming by generation market participants, and modifications necessary to conventional energy management system tools such as unit commitment. The emergence of financial tools for risk management was also pointed out in that paper. Sheblé [8] identified the emergence of the view that electricity is a commodity and would be treated as a commodity. In that paper, some unresolved issues were raised, such as transmission wheeling priorities, the obligation

to serve, transmission network upgrades, etc. From these papers, it is evident that restructuring of the energy markets in the United States is imminent, if not already occurring. It is also obvious that the restructuring will have complex and far reaching effects on power system operations.

From the customers' viewpoint, Stein [9], provided some insights into a significant driver of the restructuring of US energy markets – retail loads. In that paper, Stein argued General Motors' case for lower electricity prices by giving them the ability to choose their electricity provider.

## 2.2 Energy Markets – Implementations and Rules

This section reviews papers relevant to implementation and rules for some of the proposed competitive energy markets. Based on these papers, a subset of rules was defined in this research to implement an energy brokerage simulator.

Energy brokerages are the specific type of energy markets being considered in this research, and consist of double auctions on the part of buyers and sellers of energy. One of the earliest papers on energy brokerages is by Barker [10]. In that paper, the author reported widespread interest in energy brokering in the United States. The author noted that the energy broker (at that time) could be either the primary facilitator for interchange energy trading, or could be a supplemental market to the existing bilateral interchange markets. At this point, it is still not clear if the energy broker will become the sole facilitator of energy trades. So in this research, we assumed that there is a possibility that participants may have energy contracts outside the jurisdiction of the broker.

Several methods have been proposed for implementing brokerages. In reference [11], Doty et al proposed brokerages implemented based on high-low matching where the buy bids are ranked in decreasing order, and the sell bids are ranked in increasing order,

and the highest buy bid is “matched” to the lowest sell bid, and so on until no further savings are possible. A linear programming based implementation was proposed by Fahd et al in [12], where the optimal bids were determined by sensitivity analysis on economic dispatch, followed by LP-based bid matching. An optimal power flow (OPF) based implementation was proposed by Fahd et al in [13], where the optimal bids were determined by an OPF solution, followed by LP-based bid matching. The advantage of this method is that network constraints can be incorporated. An augmented Lagrangian-based approach was presented by Anwar et al in [14] for implementing the OPF. The amount of energy for sale and purchase were calculated in that paper by parametric analysis on the OPF solution.

A three-level approach was presented by Post et al in [15]. In that paper, the first level consisted of optimally allocating supply and demand independent of the transmission network. The novel aspect of this allocation was the use of reservation prices by the sellers. This was followed by adjustment of supply allocation for losses. This was done by calculating losses by using penalty factors, and increasing the generation of the most economical generator to compensate for this loss power. The third step was an LP-based transshipment problem solution, with capacity constraints considered, to maximize the surplus from transactions that were feasible with respect to network conditions. This was modeled as a minimization of transmission usage costs using the MW-mile method. Also, it was assumed that FACTS devices were in place that controlled the flows on the lines. Thus, the implementation of energy brokerages has been fairly well researched.

Another key aspect of brokerages is a “fair” allocation of the resultant savings. In [16], a method that allocates savings based on the Shapley value of the participant is presented by Chattopadhyay. Shapley values are an estimate of the worth of a participant in an interconnected system from the point of view of net savings, and hence are expected to be superior criteria upon which to base allocations. Herriott [17] established theoretical grounds for equitable allocation of savings in the brokerage. In [18], Hahn

et al examined experimentally, the relative efficiencies of the split-savings method of allocation, and the single market-price method of allocation. The author noted that the single market-price showed higher efficiencies; however, both methods showed at least 90% efficiency as compared to the competitive equilibrium.

The advent of deregulation imposes some additional considerations on the energy brokerage problem. The objectives of the participants in the brokerage now becomes profit maximization, as opposed to cost minimization, even though the objective of the broker remains the same. In [19], Sheblé et al pointed out that several regulatory questions need to be answered, such as wheeling costs, loss allocation, priority of transactions, etc. The answers to these questions are still not clear.

The study of auction mechanisms in organized markets by experimental methods has been a well researched subject. An acknowledged leader in the area of experimental economics applied to auctions is Dr. Vernon Smith. An example of this work can be found in the paper by McCabe, Rassenti, and Smith [20], wherein the authors presented results from a computer-assisted market that were simulated in a laboratory environment. These markets deal with gas and electric power, and use auction mechanisms to implement a competitive environment. In that paper the authors pointed out that, in these markets, the individual generator owners might submit bid prices that are lower than marginal generating costs, because of startup/shutdown cost considerations. The authors also suggested that spot prices at the buses be used to determine transmission prices. In [21], Sheblé identified details for implementation of energy brokerages in the open transmission access context. In that paper, the author proposed the analysis of electric energy as a commodity in a financial framework. Definitions were provided for contracts traded in the cash market, futures market, options market, clearing house market and planning market. Another model was presented by Kumar et al in [22] that considered an auction game. The objective of the auctioneer was to determine an optimal schedule of power transfer. This model assumed multiple rounds of bidding for

each period, where the auctioneer could continue to request additional bids from sellers and buyers until price discovery has taken place. The auction mechanism was assumed to be the high-low matching method. The advantage of multiple rounds per period is that participants have a better chance to discover the correct state of the market and respond to that state, rather than base their strategies only on historical data. This presumably improves the chances of clearing the market (satisfying the demand with the supply).

Alvey et al [23] described a security-constrained bid-clearing system being used in the New Zealand wholesale electricity market. The model described in that paper is similar to the models described in the other paper reviewed in this section, in that the auction mechanism is a sealed double auction. The price used, however, is a single market clearing price, defined as the price of the last bid cleared. Also, an LP algorithm was used to solve the network constrained optimization problem, with the nodal prices determined by LP. In recent work, Singh et al [24], studied the effects of using a location dependent nodal spot pricing scheme, as opposed to using a single market clearing price mechanism. The authors showed in that paper that the former method could lead to arbitrage opportunities for the suppliers.

Other models that have been proposed include the England and Wales Power Pool system, where the bids submitted by the participants also include unit commitment related data, in addition to bid prices, such as ramp rates, unit minimum run and down times, etc. In [25], Jia and Radinskaia showed that the heuristics used to implement the bidding rules in England can be derived analytically, using Lagrangian relaxation. However, Oren et al [26] have shown that, from the implementation point of view, the Lagrangian relaxation-based unit commitment algorithm used to implement such a competitive market is subject to volatility in the price signals provided and the profitability of some units, depending on the parameters for step size selection used in the dual optimization. Thus, that paper provided support for a more decentralized approach to

the energy market, where a simpler auction is performed, with details of unit commitment and scheduling left to the individual participant to determine. In [27], Al-Agtash showed a Lagrangian multiplier based implementation for the operational planning in such a system, which is a simplified model of the proposed California power exchange (PX) and independent system operator (ISO).

In this research, we have assumed that the decentralized model (somewhat similar to the proposed California market model) will be in effect, and that unit commitment decisions will not be the concern of the broker, but of the individual participant. As far as the broker is concerned, each hour will be treated separately from the other, in other words, there is no temporal constraint in the objective of the broker.

Another key consideration that is a result of open access to transmission is the pricing of ancillary services. In [28], Kumar et al presented the approach that every transaction in an energy brokerage depends on multilateral contracts for ancillary services. A separable programming problem with a piecewise linear objective function is presented as the objective for the broker to maximize. The objective translates to maximizing transactions while ensuring system demand, spinning and ready reserve requirements, and satisfying the transmission line losses.

Based on a literature review, some important assumptions have been made in this research. These are summarized as follows. We implemented a simplified brokerage model that follows the high-low matching algorithm that is outlined in [11]. This was followed by a power flow based algorithm outlined later, that attempts to maximize surplus from feasible transactions. This is essentially a simplified variation of the model presented in [15].

Savings allocations are determined by the split-savings method given in [11], even though this may not be a “fair” allocation under all scenarios. The reason for this is the ease of implementation and the current use of split savings in the power industry. In the context of reference [21], this research deals only with the cash market. Inasmuch as

transactions may occur after a certain time period from price determination, the model in this research might be classified as a forward market, but the classification is not central to any of our approaches. Single round (or one-shot) bidding for energy has been the norm in the brokerage markets implemented or proposed in several instances ([10], [11], [16], [18]) and was the model assumed in this research as well. But the strategies developed are by no means irrelevant in the multiple round scenario. Indeed, the strategies developed in this research can be envisioned as starting points for developing bids in each of the several rounds in each period of bidding, if a multiple round model is assumed.

Ancillary services are assumed to be contracted outside the jurisdiction of the broker. However, it is recognized that future strategies will be affected by and must consider the level of usage of ancillary services by the participants. At this point in time, ancillary service considerations are beyond the scope of the model proposed here.

### **2.3 Strategic Bidding – Other Industries**

The papers reviewed in section 2.2 deal with implementing energy brokerages with the goal of optimizing total system savings, subject to a variety of constraints, including non-traditional considerations such as ancillary services. This was usually followed by equitable allocation of savings according to certain rules. Thus the papers discussed the structure of the market. Within this structure, participants choose to bid for energy based on different factors. If factors other than just the cost of production are considered, then the participants are said to bid strategically. Strategic bidding is the process by which a participant in a market develops bids that are perceived to be effective in achieving its performance goals. Such strategic bidding behavior, while uncommon in an energy brokerage context, is common in other industries. Stark et al [29] presented a comprehensive bibliography on competitive bidding. The number of citings in that

reference shows that competitive bidding and the area of strategic bidding in particular, are very well researched in other industries. The following sections review literature pertaining to strategic bidding, both in the power industry and in other industries.

Some of the industries where strategic bidding is commonly used include construction, oil tract leasing, and property sales. Such bidding typically relies on market information, such as transaction prices to estimate competitors' behavior. Based on this market information bids are developed. One of the earliest papers on competitive bidding is by Friedman [30]. In that paper, the author suggested a variety of objectives that a company could use in competitive bidding. The author also presented a method to estimate the probability of winning a bid against competitors based on market data, and to use this probability to determine the optimal bid. In reference [31], Griffis provided a method by which competitors are modeled according to probability distributions of their bids. This same reference [31] also presented a method for updating probability distributions when new data become available. In reference [32], Baron discussed the effect of risk aversion on optimal bid prices of a firm in the case of incentive contracts. Wilson [33], discussed the equilibrium solution in the case of asymmetric information among competitors. In reference [34], Lavalle presented a Bayesian approach to bid development, based on conditional probabilities, involving asymmetric information and asymmetric perception on the possession of such information. In [35], Skitmore et al proposed a multivariate approach to using the probability distributions, thus extending the methods in works such as [30]. In reference [36], Boughton presented approaches to bidding that go beyond probabilistic models. Some of these methods may be applied directly to energy brokerages. Other methods may be modified (such as by using the lower bounding method presented later in this research) for application in energy brokerages.



## 2.4 Strategic Bidding – Power Industry

This section reviews some of the papers that apply strategic bidding techniques in the power industry context. In [37], Rozek provided a survey on developments in competitive bidding in the emerging electricity markets. That paper primarily focused on the new generation capacity market, rather than the energy market. Nevertheless, some of the issues are the same for the energy markets. The issues, that were raised in that paper, include the administration of the bid evaluation process, influence of the bidding rules on the offer strategies of the bidders, and the use of experimental economics, i.e., markets simulated in a laboratory environment to study the effects of various factors on market performances. In [38], Kahn et al presented optimization methods for evaluating competitive private power contracts in the capacity planning function of a utility. That paper presented the use of the Benders decomposition approach to select the optimal bids. The paper assumed that the bids are for dispatchable blocks of power submitted by different producers (sellers) to the utility (sole buyer) for consideration. Thus that strategy considered the case of a dominant buyer who has complete knowledge of the market price distribution in advance.

In [3], David presented a strategic bidding approach for the case where there are a number of sellers, and one buyer, who picks the least bid seller. Both a deterministic and a probabilistic formulations were presented. The approaches in that paper are similar in the following ways to the approaches in this research. The probabilistic approach in both involved using distributions for the bids of competitors. But no mention is made in [3] of what distributions should be used to model competitors, and how the strategies are implemented for more complex distributions than those with a linear CDF. In this research, we investigated both issues in detail. Both methods have as an objective, the maximization of expected value of profit. However, that paper considers a very simple market structure, neglecting the possibility that buyers might also be

submitting a variety of bids. Also, minimum up and down times of units, and unit commitment costs. These are components that have been included in our research, in the illustrative examples presented in Chapters 5 and 6. Also, while the author of that paper has commented on using utility functions to model preference functions, no analysis is presented on the nature of the utility functions that could be used, and the effects of risk attitudes on bidding strategies. These issues have also been investigated in this research<sup>1</sup>.

The method in [3] has one advantage in that, the objective function is posed as a dynamic programming problem, with the stages being the bid blocks, and the states being the bid prices. Since dynamic programming is very flexible in possible modifications to objective function, this might be a worthwhile consideration in future additions to this research.

In [39], Richter has presented a novel approach to strategic bidding in the brokerage market using genetic algorithms (GAs). The brokerage model assumed is similar to that given in [22], with multiple rounds of bidding for each period. Richter used GAs to both predict the future market price, as well as determine the optimal bid. This was done by evolving genetic agents that perform better in each round of bidding. Also, that work was presented as the development of a *framework* for bidding strategies. It would be interesting to see further developments in that area when complex strategies are actually analyzed. It would be of value to see if the strategies developed by using intelligent agents could be distilled in the form of easy to understand rules. Also conceivable is the scenario that participants may not place a high premium on knowing *why* a particular agent performed well in a given situation, so long as it is possible to reproduce the bidding performance in a market situation.

---

<sup>1</sup>While a part of this research was performed after the publication of reference [3], our work was independent of the work presented there. In fact, application of probability distributions to model competitors is an idea already applied widely in other industries, as shown in Section , and those references were the primary source of inspiration for our work.

The primary difference between [39] and this research is that this research considers a framework that includes unit commitment considerations and, to a limited extent, transmission considerations. However, this research does not explore the use of genetic algorithms for bidding. Instead, conventional methods for optimal bidding are used with the application of expected value or utility maximization.

In recent work, Sakk et al [40] reported the use of a neural-network based learning algorithm to develop bidding strategies in a sequential bidding model. The strength of this algorithm is that it being implemented as a tool that can be used across the internet, with no additional capability on the part of the users except a web-browser. However, in that paper, it was assumed that bidding histories of the participants were made public. This may not be a realistic assumption, since this could reveal cost structures of the participants. In [41], Hao et al formulated a bidding model based on consumer payment minimization. This is a relatively new approach, compared to the unit commitment-like methods that aim to minimize total system costs, or maximize total system savings. Under this model, the bidding strategy of the units that are “on the margin”, i.e., with operating costs close to the market clearing price, is expected to be to slightly shave the bid, in other words to bid slightly above the marginal cost, for selling. However, it is unclear as to what the exact amount of shaving should be. Also, buyer side bidding is not considered.

An interesting paper related to strategic bidding was presented recently by Krishna [42], where intelligent agents (computer programs) were allowed to negotiate and communicate with intelligent agents of other players, to form collusive coalitions in market games. This is a novel approach based on game theory. In our research, however, we do not assume that participants form any coalitions.

## 2.5 Generation and Transactions Scheduling

In this section, transactions and scheduling related literature is reviewed. In the past, some approaches have been tried wherein the uncertainties in system scheduling data are incorporated in scheduling transactions. In [43], Prasannan et al presented a method to incorporate sale transactions in the hydrothermal coordination problem. In that paper, the authors assumed that transaction prices are predetermined by the selling utility and power levels at this price are offered to the purchasing utility. This purchasing utility might then choose up to the level of power offered at the offer price. The method presented is an attempt to optimize this real time pricing problem by Lagrangian relaxation. In [44], Zhang et al presented a similar approach, solved by using the augmented Lagrangian decomposition method. Both methods attempt to minimize cost. In [45], Fan et al considered including the effect of committing purchase transactions on system energy price by a first order approximation of a Taylor expansion of the system energy price with respect to transaction power. The unit commitment algorithm proposed was a sequential unit commitment method, which is essentially a heuristic method. The above references are relevant to later sections where scheduling related strategies are concerned. In those sections, the approaches that incorporate uncertainties involve quantifying the risk from the uncertainties, and modifying the bid prices based on this risk. While Lagrangian relaxation based approaches were used in the approaches, the unit commitment tool used in our research was a priority list based program that is admittedly less flexible than the Lagrangian relaxation approach. This method was chosen for ease of implementation and speed of execution. However, the strategies developed in this document are not dependent on the type of unit commitment tool used, and will be equally applicable if the participant were to use a Lagrangian relaxation based tool for unit commitment.

In [46], a stochastic unit commitment model was proposed by Takriti et al, based on

scenario bundles, for situations where the demand is uncertain. The basic concept is to minimize the probability weighted average cost of production for the possible scenarios. The solution process was by Lagrangian relaxation. In [47], Breiphol et al presented a Gauss-Markov load model for application in risk evaluation. In that paper, the load model was used to predict system hourly load mean and variance based on the previous hour's load. In [48], Hoffer et al discussed various distributions to be considered for conditional load duration curves. The distributions considered include gamma, beta and triangular. These references are relevant to the discussion of the effect of load uncertainties on the participant's bidding strategy, in later sections.

In [49], Billinton et al considered the generating system operating health and risk with and without stand-by units, interruptible load, and postponable outages. A risk index was defined to quantify the probability of the system being at risk (in this context, the sum of the probabilities of the system being at emergency state and extreme emergency state, because of unit outages.)

However, in the context of a competitive market, it may be difficult to assess an acceptable risk level based on such an index. An intuitively more appealing approach is one where scheduling options available to participants could be evaluated based upon a method that considers not only the relative profitability of selecting the option, but also upon the participant's attitude towards risk-taking. Such a quantity is the so called *utility function* of the participant. Literature relating to this area is reviewed in Chapter 6, as part of Section 6.4.2.

## 2.6 Transmission Access and Pricing

This section references some papers that are relevant to the assumptions made in this research with respect to transmission. Transmission is the most complex of the issues that were raised with deregulation; there is a large volume of literature on this

subject. This section is not to be considered by any means, an extensive bibliography. In fact, Lankford et al have published such a bibliography for the IEEE Task Force on Transmission Access [50]. In [51], Vojdani et al raised some of the issues that have to be resolved with the advent of open transmission access. Some of these include models for transmission rights, dispatch, control and pricing. The authors noted that the FERC accepted contract path as a way of pricing transmission usage. However, alternatives were expected to be proposed for this method that reflect actual flows on lines. Naumann [52] presented the impact of FERC Orders 888 and 889, which required the posting of pro forma open access tariffs, and available transmission capabilities (ATCs) on the open access same-time information system (OASIS), and also required that reservations for transmission be made only through the OASIS. In that paper, the author related the experiences of the Commonwealth Edison Company. Some of the impacts were an increase in transaction volume, increased concern about loop flows, and the complexities involved in the request, reservation and use of transmission service.

In [53] Outhred et al presented the various alternatives being considered by the Australian industry for transmission pricing. Of these, the “benefits method” (which consists of allocating the cost of the high voltage network element according to user benefit generators and loads equally), and the distance based MW-Km (or MW-mile) method were selected as the two most favored options. Based on these options, a nodal auction market model was presented.

In [54], Tabors discussed the different approaches to transmission pricing, short-run marginal cost (SRMC) based, and long-run marginal cost based (LRMC). In reference [55], Happ has described four different cost of wheeling methodologies, including the MW-mile method. In [56], Tsukamoto et al proposed the allocation of fixed transmission to wheeling transactions as an extension to the MW-mile method. The extension is the consideration of cooperative game theory to minimize the maximum regret of each participant, with the allocation of costs. In [57], Scarfone presented an interesting

application of short-circuit simulations to calculate MW-miles of a transaction. The basic concept is to perform a short-circuit study of a fault at the delivery bus being considered, by means of commercially available software, and to use a percentage of the fault current on each line to determine the MW-mile impact of the proposed transaction. Obessis et al [58], presented a review of existing cost based transmission pricing schemes, and proposed a new approach to combine the spot pricing and the embedded cost based-pricing methods.

In this research, it was assumed that transmission is modeled according to the MW-mile method. The primary reason for this is ease of implementation. Also, it is superior to postage stamp method because it includes the concept of the distance between the supply and delivery points. This assumption is also supported by the indication that the MW-mile method seems to be becoming a common method for pricing transmission in the United States.

## 2.7 New Tools and Concepts in Energy Markets

In this section, some of the new tools and concepts being used and proposed for use in the emerging energy markets are reviewed. In [59], Thomas et al presented a very detailed analysis of the different kinds of tools that will be required under three different scenarios of market restructuring. These scenarios were defined as the base case scenario, the maximum ISO scenario, and the minimum ISO scenario. In the base case scenario, the market is considered in the short-term after the FERC orders 888 and 889 become effective, with no retail access, vertically integrated utilities, and non-discriminatory transmission access being provided to all users. The second scenario assumes that the ISO performs a centralized unit commitment based upon bid prices, and also sets prices. All energy is assumed to be bought and sold through this ISO. In the third scenario, the ISO does not own any generation or transmission, and primarily is concerned with

network security.

Clayton et al [60] identified different kinds of tools required for use in system planning in the competitive environments. Flatabo [61] described some of the new tools developed or adapted for use in the Scandinavian market. These include price forecasting, risk management, trading systems, and transmission congestion and pricing software.

A class of tools, proposed for risk management in the energy markets, is a set of financial derivative instruments, such as futures and options contracts for electric energy. Ramesh [62] provided the basics of financial derivatives in energy markets. Other related references in this area include [8] and [21].

From these papers, it is clear that modifications will be needed for conventional tools, and that new tools will be needed, in order to cope with the changes that market restructuring has introduced into power system operations.



### 3 MARKET-BASED STRATEGIES

#### 3.1 Perfect Competition

In a perfectly competitive market there are no barriers to entry and exit and complete and accurate information is available to all the participants [63]. In such a market, no single participant will have the market power to influence the price of the commodity being bought and sold, and will thus be a “price taker”. In such a market, the following condition can be derived for a profit maximizing producer:

$$\pi(q) = pq - C(q) \quad (3.1)$$

where:

$q$  is the quantity of the product that the participant produces,

$p$  is the market price for the commodity,

$\pi(q)$  is the profit of the producer from producing quantity  $q$  of the product,

$C(q)$  is the production cost of the producer, a function of quantity alone.

In Equation 3.1, the only production decision that the producer needs to make is that of the quantity to produce,  $q$ . Differentiating the above equation with respect to  $q$ , and applying first order conditions yields the following expression:

$$\pi'(q) = p - C'(q) = 0 \quad (3.2)$$

Rearranging the terms of the above equation yields the following well known conditions for price in a competitive market:

$$p = C'(q) \quad (3.3)$$

Equation 3.3 states that the optimal quantity that a producer should produce in a perfectly competitive market would be such that the difference between marginal cost of production and market price is zero. From a power industry perspective, this is the rationale behind moving to a price-based market structure from a cost plus-based market structure. The implication of this equation is that each producer will dispatch their generating units such that the incremental cost of production (marginal cost,  $C'(q)$ ) is equal to the market price. This, of course is the “equal Lambda” criterion that centralized economic dispatch achieves.

In the context of the energy brokerage market, this would imply that there is no need for any strategies to bid in the market. Merely bidding the marginal cost of production at each bidding period would be the optimal bidding strategy in a perfect market. However, the real energy markets that have existed so far, and are currently evolving, are not perfectly competitive markets. In the following section, some of these imperfections are examined.

### 3.2 Imperfect Market Conditions in Energy Markets

In the energy markets that have been in existence until now, and in some of the markets evolving in the future, some barriers to entry and exit exist. One barrier to *entry* is the requirement for memberships in certain power pools, power exchanges or some similar market structure. Thus, new entrants to the market cannot freely participate without investing some amount of resources as sunk costs, capital or membership fees. Also, the obligation to serve load has not been entirely removed in these markets. In the United States, one such evolving market at the time of this research is the proposed California power exchange. Thus, there is a barrier to *exit* the market. This prevents, at least to a certain extent, market participants from choosing not to generate power if market conditions such as price are not favorable. Also, the existence of large power

generating companies with a diverse mix of generating resources of varying costs and efficiencies, in conjunction with relatively weaker competitors with high cost resources suggests that there is the possibility that one or more players might have more market power than others. Another consideration that is important is the availability of information. Even though certain relevant system data is made public in the energy markets, the ability to transform this data quickly into usable information, will depend on the resources available to individual participants. The lack of liquid markets for hedging price and quantity risks also means that there is considerable uncertainty that the producer faces prior to making production decisions. Thus, we can see that in the incipient stages of deregulation in the United States, energy markets have attributes that would not conform to the perfectly competitive markets assumed in the analysis of Section 3.1.

Regulators are striving to correct such imperfections in current and proposed market structures, and might well succeed in eliminating them. However, even if future markets evolve that are very close to rectifying the above imperfections, uncertainties and risk attitudes would make it likely that participants would be reluctant to bid their marginal costs, and would instead add a profit margin to their bids. Given this situation, each participant must develop a strategy by which bids for the energy markets can be developed. In Section 1.1.3, some the factors that might affect the development of such bids was outlined. In the following sections, one such factor, modeling competitor behavior, is examined in detail.

### **3.3 Modeling Competitor Behavior**

Some attempts have been made in the literature to model competitor behavior by using game theoretic models, intelligent agents, or by conjectural variations in oligopoly models. In this research, a probabilistic approach for modeling competitor behavior is examined. In this approach, the bids of the competing participants of an energy bro-

kerage are assumed to be distributed randomly. Then, based on this distribution, the probability of success of a participant's bidding strategy is evaluated. Next, the participant's bidding strategy can be developed as an expected value maximization approach. This procedure is outlined in this section, using a simple triangular distribution, for illustration, after which, the procedure is developed for three different distributions in the following sections.

### 3.3.1 Using Bid Distributions – An Illustrative Example

Let  $p_b$  \$/MWH represent a buyer's bid price for buying a unit block of energy. Let  $c$  represent the marginal cost of generating this unit block of energy if the buyer owns generation, or the value of this unit block to the buyer. In order to achieve a goal of expected profit maximization, the buyer must maximize the following objective function:

$$\underset{p_b}{\text{Maximize}} \quad S(p_b) \left( c - \frac{p_b + p_s}{2} \right) \quad (3.4)$$

where:

$S(p_b)$  is the probability of success of the bid  $p_b$

$p_s$  is the sell bid to which the buy bid was matched.

A bid may be for multiple unit blocks of energy. Regardless of the quantity of energy involved, the blocks of energy in a single bid are all considered to be priced by the bidder on a per unit basis, at the given bid price  $p_b$ .

The competition for this bid may be represented in the form a distribution of competing buy bids. In order to be successful, the buyer's bid  $p_b$  should be greater than the competing bids. The expression for expected profit cannot be directly maximized with respect to the decision variable  $p_b$  because the buyer does not have any information, in general, on the sell bids, and in particular, on which sell bid would be matched to  $p_b$ . However, according to the brokerage rules, the sell bid  $p_s$  to which this winning bid would be matched, must be less than  $p_b$  in order for a saving to exist. Therefore, the

following relationship can be derived:

$$p_s < p_b \Rightarrow c - p_b < c - \frac{p_b + p_s}{2} \quad (3.5)$$

The second part of Equation 3.5 implies  $(c - p_b)$  is a lower bound to the profit from a successful bid to buy. The advantage of this equation is that a lower bound to expected profit from a bid can be derived as follows:

$$\underset{p_b}{\text{Maximize}} \quad S(p_b)(c - p_b) \quad (3.6)$$

Maximizing the above equation has the effect of maximizing the *lower bound* to expected profit. This in general need not result in maximum expected profit. Therefore, the strategy resulting from maximization of the lower bound to expected profit is considered to be a “suboptimal” bidding strategy, because of the lack of seller information.

### 3.3.2 Development of a Suboptimal Bidding Strategy

In this section, an illustrative case is presented to explain how a participant could develop a suboptimal bidding strategy, as defined in the previous section, using the available market information. First, for this illustrative case, we will assume that the participant has available a subjective probability distribution to represent all the competing bids for a given hour, and that this is a triangular distribution as shown in Figure 3.1. This shape is only assumed here for illustrative purposes. In the following sections, the strategy is developed for three other shapes, that are more representative of a practical example, and the shapes are obtained by fitting the distributions to data from simulations. However, the simple triangular shape will allow us to derive a closed form expression for the suboptimal bid, and illustrate the mechanics of the strategy easily.

Let us assume that the participant, who is considered a buyer, uses a probability density function (PDF) as shown in Figure 3.1.

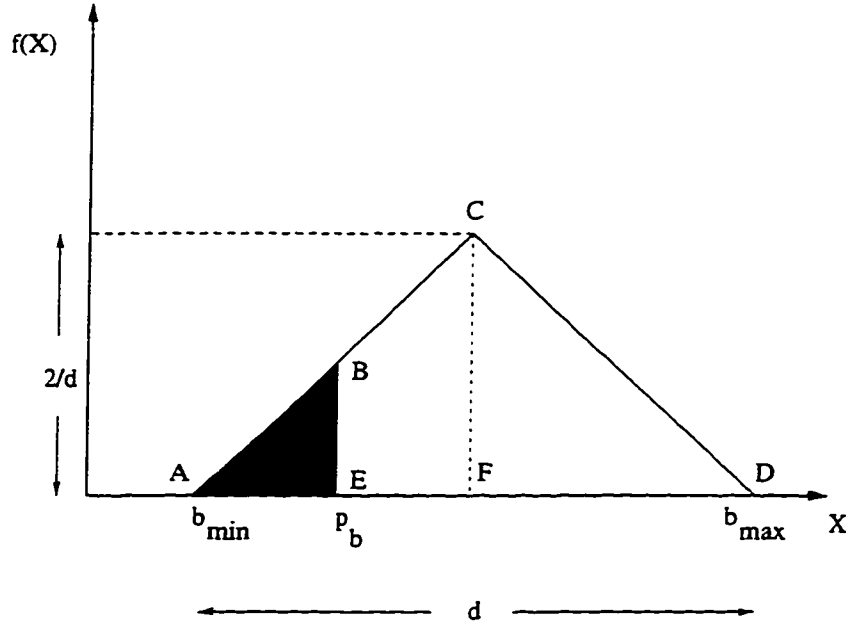


Figure 3.1 Triangular PDF for buy bids.

In this figure, the random variable  $X$  represents the competitors' buy bid. The quantities,  $b_{min}$  and  $b_{max}$ , represent values below and above which the competitors are not expected to bid. The distance  $d$  between  $b_{min}$  and  $b_{max}$  represents the possible spread over which the competitors are expected to bid. The probability density function for  $X$  is  $f(X)$ . By definition, the area within the triangle  $ACD$  is one. Further, let us assume that<sup>1</sup>  $AF = FD = \frac{d}{2}$ . For a given buy bid value of  $X = p_b$ , the cumulative distribution function (CDF) of competitors' bids,  $F(p_b)$  is given by:

$$F(p_b) = \int_{-\infty}^{p_b} f(X)dX = Pr(X \leq p_b) \quad (3.7)$$

In other words, the CDF evaluated at  $p_b$  gives the probability that the competitors will bid a value less than or equal to  $p_b$ . Thus, if the buyer bids a value just greater than  $p_b$ ,  $F(p_b)$  gives the probability of winning the block of energy over his competitors. Thus,  $F(p_b)$  gives the probability of success of a bid  $p_b$ . This was denoted by  $S(p_b)$  in Equation 3.4. For the triangular PDF,  $S(p_b)$  is the area of the region  $ABE$ . It can be

<sup>1</sup>Such a symmetrical distribution is assumed only for illustrative purposes. In reality, participants could assume asymmetrical distributions.

shown that this area is given by:

$$S(p_b) = \frac{2(p_b - b_{min})^2}{d^2} \quad (3.8)$$

if  $p_b \leq b_{min} + \frac{d}{2}$ , and by:

$$S(p_b) = 1 - \frac{2(b_{max} - p_b)^2}{d^2} \quad (3.9)$$

if  $b_{min} + \frac{d}{2} \leq p_b \leq b_{max}$ .

The lower bound to expected profit from bidding  $p_b$  is obtained by substituting this expression for  $S(p_b)$  in Equation 3.6 and is denoted by  $E_{lb}(p_b)$ :

$$E_{lb}(p_b) = S(p_b) \cdot (c - p_b) \quad (3.10)$$

Applying the first-order necessary conditions to  $E_{lb}(p_b)$ , we can solve for the suboptimal buy bid price,  $p_b^*$ . This is done by taking the first derivative of the expression given by Equation 3.10 with respect to  $p_b$ , and setting it equal to zero. Since we have two expressions for  $S(p_b)$ , we must solve both forms of the first-order necessary condition. Then, depending on where the solution for  $p_b$  lies, to the left or the right of point  $F$  in Figure 3.1, the solution obtained from using the corresponding form of  $S(p_b)$  should be selected. The closed-form solutions for the suboptimal bid for the two possibilities are given by the following equation:

$$\begin{aligned} p_b^* &= \frac{2c + b_{min}}{3} \\ &\text{if } p_b^* \leq b_{min} \leq \frac{d}{2} \\ p_b^* &= \frac{4b_{max} + 2c - \sqrt{(2b_{max} + c)^2 - 6(2b_{max}^2 + 4cb_{max} - d^2)}}{6} \\ &\text{if } b_{min} + \frac{d}{2} \leq p_b^* \leq b_{max} \end{aligned} \quad (3.11)$$

The second-order sufficient condition is obtained by differentiating Equation 3.10 twice w.r.t.  $p_b$ , and setting the resulting expression to be less than zero. This is satisfied

if  $c > b_{min}$ . This condition translates to the requirement that the marginal generating costs of the buyer be greater than the lower limit of the competitor's bid. If  $c < b_{min}$ , the solution for  $p_b^*$  is  $c$  itself. This gives a zero probability of acceptance. In other words, if the buyer can generate power at a lower cost than the competitor's lowest buy bid, it is not worth bidding more than marginal cost. On the other end of the spectrum, if  $p_b^* > b_{max}$ , the buyer should not bid a price value greater than  $b_{max}$ , since the probability of acceptance is not increased beyond the probability of acceptance at  $b_{max}$ ,  $[S(b_{max}) = 1]$ . Thus, for  $c < b_{min}$ , the suboptimal bidding strategy described dictates that the buyer should bid according to Equation 3.11, upto a maximum of  $b_{max}$ . The corresponding values of the probability of acceptance,  $S(p_b)$ , and the expected value of the lower bound on the savings,  $E_{lb}(p_b)$ , are obtained by substituting this value for  $p_b^*$  in Equations 3.8, 3.9 and 3.10.

Similar arguments can be applied to the case of the participant who is considered a seller. For a seller to be successful, the selling bid  $p_s$  must be the lowest bid for a given block of energy. Thus, the probability of success of a sell bid  $p_s$ ,  $S(p_s)$  is given by the following equation:

$$S(p_s) = Pr(p_s < X) = 1 - Pr(X \leq p_s) \quad (3.12)$$

Figure 3.2 shows such a triangular distribution for a seller. Again, the area of the triangle ACD is 1 by definition. The complement of the CDF evaluated at  $p_s$  is shown by the shaded area, region BED.

In other words, the probability of success is the *complement* of the probability that the seller's bid  $p_s$  is higher than all the competing bids. But the rightmost term in the above equation is simply the CDF of competing sell bids, evaluated at  $p_s$ . Thus, the probability of success for a seller can also be related to the CDF of the competing sell bids, in a manner very similar to that for a buyer. Closed form expressions for a seller's suboptimal bid could also be derived.



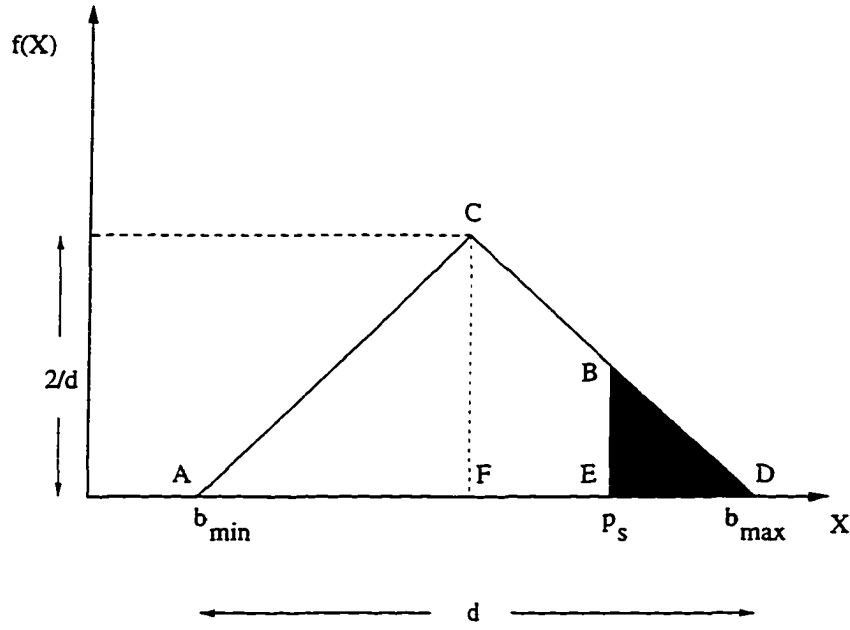


Figure 3.2 Triangular PDF for sell bids.

### 3.3.3 A Numerical Example

In this section, the mechanics of the suboptimal bidding strategy for the triangular distribution are illustrated by means of a numerical example. Let us assume that the buyer's notion of the competitors' bidding behavior corresponds to the values for  $b_{min}$  and  $b_{max}$  in Table 3.1.

Let us also assume that the marginal cost of generation of the buyer varies according to the values for  $c$ . Then, for each value of  $c$ , the corresponding value of the suboptimal bid, the probability of acceptance, and the maximum expected lower bound on profits are given in Table 3.1, based on the first part of Equation 3.11, and the following reasoning where the equations yield unacceptable values. Cases 1 and 2, where  $c = 8$  and  $c = 10$  correspond to cases where the buyer's generating costs are lower than or equal to  $b_{min}$ .

For these cases, the suboptimal strategy dictates that the buyer should not bid above the marginal cost of generation. Thus, the suboptimal bid is the marginal cost itself. The probability of acceptance for these cases is zero. In Cases 3 and 4, where  $c = 12$  and

Table 3.1 Variation of suboptimal values with marginal cost – triangular distribution.

Case	$b_{min}$	$b_{max}$	$c$	$p_b^*$	$S(p_b^*)$	$E_{lb}(p_b^*)$
1	10.00	15.00	8.00	8.00	0.0000	0.0000
2	10.00	15.00	10.00	10.00	0.0000	0.0000
3	10.00	15.00	12.00	11.33	0.0711	0.0474
4	10.00	15.00	13.75	12.50	0.5000	0.6250
5	10.00	15.00	14.00	12.60	0.5386	0.7540
6	10.00	15.00	16.00	13.26	0.7592	2.0802
7	10.00	15.00	18.00	13.72	0.8704	3.7253
8	10.00	15.00	25.00	14.43	0.9735	10.2899

$c = 13.75$ , the suboptimal strategy yields a positive probability of acceptance  $S(p_b^*)$ , and an increasing maximum lower bound on expected profit  $E_{lb}(p_b^*)$ . Also, for these values of  $c$ , the solution for  $p_b^*$  lies to the left of point  $F$  in Figure 3.1, so we choose Equation 3.11 to determine the suboptimal bid. In Cases 5 through 8, where  $c = 14$ ,  $c = 16$ ,  $c = 18$ , and  $c = 25$ , respectively, the solution for  $p_b^*$  lies to the right of point  $F$  in Figure 3.1, so we choose the second part of Equation 3.11 to determine the suboptimal bid. These considerations are illustrated in Table 3.1. It can be seen that as the suboptimal bids increase in price, so do the corresponding probabilities of acceptance and lower bounds to expected profits.

These values are implicitly based on two assumptions. The first assumption is that the buyer's goal is to maximize expected value of savings. The second assumption is that the buyer has a notion that the competitor's bidding behavior will obey a triangular density function. The former assumption could be varied to fit various performance goals. For example, if the participant is a utility that is only interested in keeping a certain plant shut-down, regardless of the cost of energy, then the goal could be to maximize probability of acceptance, instead of maximizing expected value of savings. The latter assumption admittedly makes the problem easier to solve in the closed form. However, if the buyer assumes a more complex density function, the general principles of the

strategy still can be applied, with a higher degree of computational intensity. In the following sections, three such functions are explored.

### 3.4 Polynomial Modeling of a Bid Distribution

In this section, a suboptimal bidding strategy based on directly modeling the CDF of competing bids is developed. This is because of difficulties in fitting a model to the PDF, followed by the numerical complexities in integrating this PDF to obtain the CDF. A CDF is usually smoother than a PDF, and hence is easier to fit a model to. An approximate CDF can be obtained by constructing a cumulative relative frequency (CRF) curve from the histogram of historical transaction prices. Fitting a polynomial curve to this CRF curve then yields an expression for the CDF as function of bid price. Figure 3.3 shows an example where a polynomial of degree 5 is used to fit a CRF curve obtained from historical transaction prices. These historical prices are obtained from simulations performed on a brokerage simulator described in Chapter 4. A polynomial of degree  $n$  is assumed in this section, to represent the CDF of competing bids.

Now, following similar arguments as that presented in Section 3.3, we equate the probability of acceptance,  $S(p_b)$  to be the CDF of the competing bid distribution,  $F(p_b)$ , evaluated at  $p_b$ . The objective now becomes:

$$\begin{array}{cc} \text{Maximize} & \\ p_b & F(p_b)(c - p_b) \end{array} \quad (3.13)$$

Let us assume that the result of fitting a polynomial of degree  $n$  to the CRF curve results in the following equation for CDF of competing bids:

$$F(X) = a_0 + a_1X + a_2X^2 + \dots + a_{n-1}X^{n-1} + a_nX^n \quad (3.14)$$

The competing bids are represented by the random variable  $X$ , and the coefficients of the polynomial in Equation 3.14 are obtained by regression. For a buyer,  $F(p_b)$  will

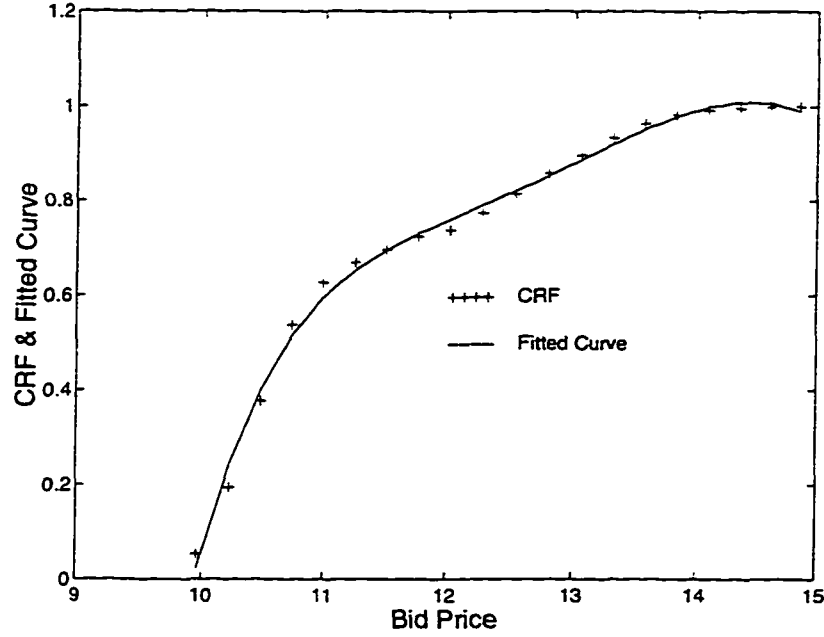


Figure 3.3 Fitting a polynomial to bid data.

represent the probability of success of bid  $p_b$  following the arguments of Section 3.3.2. Thus, the following expression can be derived for the lower bound to expected profit from bid  $p_b$ , in the case of a buyer using the polynomial model:

$$E_{lb}(p_b) = S(p_b)(c - p_b) = F(p_b)(c - p_b) \quad (3.15)$$

Using Equation 3.14, this leads to:

$$E_{lb}(p_b) = (c - p_b)(a_0 + a_1 p_b + a_2 p_b^2 + \dots + a_{n-1} p_b^{n-1} + a_n p_b^n) \quad (3.16)$$

Applying first order necessary conditions to the above equation will result in the following expression<sup>1</sup>:

$$\begin{aligned} & (n+1)(ca_{n+1} - a_n)p_b^n + n(ca_n - a_{n-1})p_b^{n-1} + \\ & \dots + 2(ca_2 - a_1)p_b + (ca_1 - a_0) = 0 \end{aligned} \quad (3.17)$$

<sup>1</sup>In Equation 3.17, the coefficient  $a_{n+1}$  is a fictitious coefficient introduced only for reasons of symmetry. It is set to zero before the equation is solved. The advantage is that the coefficients of the first order condition can now be expressed by a general formula, which helps in computer implementation.

The derivation of the above expression is given in Section A.1.1. The solutions of Equation 3.17 that satisfy second order sufficient conditions, and the condition  $p_b < c$  (the buyer should not bid a value greater than generating cost) will be the desired suboptimal bids. In order to find the best bid in case of multiple solutions for Equation 3.17, the participant can simply pick the bid that results in the maximum expected lower bound to profit that result from Equation 3.16.

A similar approach may be followed by a seller using a polynomial model. In this case, the probability of success of a sell bid  $p_s$  will be given by the following equation:

$$S(p_s) = Pr(X > p_s) = 1 - F(p_s) \quad (3.18)$$

In other words, the seller must have the lowest bid compared to his competitors. Also, the lower bound to expected profit will now be given by

$$E_{lb}(p_s) = S(p_s)(p_s - c) = (1 - F(p_s))(p_s - c) \quad (3.19)$$

With these modifications, it can be shown that the equation for suboptimal seller's bid is given by<sup>2</sup>:

$$\begin{aligned} (n+1)(ca_{n+1} - a_n)p_s^n + n(ca_n - a_{n-1})p_s^{n-1} + \\ \dots + 2(ca_2 - a_1)p_s + (ca_1 - a_0 + 1) = 0 \end{aligned} \quad (3.20)$$

Thus, the participant can build a realistic model of competing bids by polynomial modeling, and incorporate this model in the bidding decision easily. The advantage of the polynomial model is the ubiquitous nature of polynomial regression modules in most spreadsheet packages. So obtaining such a model from historical data is relatively simple. Another advantage is that for lower degree polynomials, a closed form expression for suboptimal bids can be derived from the first-order necessary conditions. Thus, the polynomial model is a good candidate for modeling CDF of competing bids.

---

<sup>2</sup>Here  $a_{n+1}$  is again zero, and was included for assistance in computer implementation.

Table 3.2 Variation of suboptimal values with marginal cost – polynomial distribution.

Case	$c$	$p_b^*$	$S(p_b^*)$	$E_{lb}(p_b^*)$
1	8.00	8.00	0.0000	0.0000
2	10.00	9.97	0.0325	0.0010
3	12.00	10.70	0.5020	0.6533
4	14.00	11.12	0.6279	1.8034
5	15.00	11.30	0.6616	2.4490
6	17.00	11.63	0.7123	3.8246
7	17.50	11.73	0.7246	4.1838
8	20.00	13.13	0.8968	6.1605

Some numerical examples are given in Table 3.2 to illustrate the variation of suboptimal values with the cost of generation, for the case of a buyer. The values given in Table 3.2 are based on the assumption that a polynomial fit is obtained by fitting a fifth degree polynomial to a sample of data that represents an approximation to competing buy bid distribution. Although the values used for  $c$  are the same as in Table 3.1, the resulting suboptimal bids are different. This is because the distribution parameters in this case, are obtained from curve fitting a set of data, and not assumed as in the case of the illustrative example for the triangular PDF.

Let the curve fit result in the following equation for  $F(X)$ :

$$F(X) = 0.0013X^5 - 0.0930X^4 + 2.5674X^3 - 35.1178X^2 + 238.4708X - 643.4213 \quad (3.21)$$

The sample data curve and the fitted curve are as shown previously in Figure 3.3. Details on obtaining such a sample and results of simulations based on this method are given in Chapter 5. Then, by using the approach developed in this section, the buyer's suboptimal bids can be computed based on Equation 3.17. These are shown in Table 3.2. It can be seen from this table that for very low costs of generation, such as 8 \$/MWH, the best bid is the same as the marginal cost. This is because the suboptimal bid for

this cost by polynomial modeling yields a value for suboptimal bid that does not satisfy  $p_b < c$ . In other words, such a bid would result in a negative lower bound on expected profit. Thus, for this cost, the best value to bid would be the generating cost itself. The table shows a zero probability of acceptance, and zero expected profits corresponding to this bid. But for higher values of generating cost, it can be seen that the procedure results in suboptimal bids that lead to higher probabilities of acceptance (approaching 1) and higher lower bounds to expected profit (approaching  $c - p_b^*$ ).

The disadvantage of the polynomial fit is that for a good fit, a higher degree polynomial is required. This leads to multiple solutions that must be evaluated. Also, it is difficult to obtain a set of coefficients for higher degree polynomials, that satisfy the requirement of the CDF that the value of  $F(X)$  be between 0 and 1. In addition, the PDF of a polynomial CDF is also a polynomial. In general, the shape of such a PDF is hard to interpret, and might not resemble a normal distribution. While this is not a serious problem, there exist some other distributions that are better candidates than the polynomial distribution from these points of view. Two of these are investigated in the following sections.

### 3.5 Incomplete-Beta Function Modeling of a Bid Distribution

In this section, the CDF of competing bids is modeled in the form of an incomplete-beta function, whose shape is defined by two shape parameters. In other words, the competing bids are assumed to be distributed according to a beta distribution. The beta distribution has been used to fit distributions whose range of variation is known [64]. The mean of the distribution depends on the ratio of its shape parameters. The variance of the distribution is inversely proportional to the magnitude of its shape parameters. As this magnitude increases, the distribution tends towards a normal distribution. The following analysis defines and develops the incomplete-beta function as a possible way

to model competing bids.

The expression for the Beta function is defined as follows [65]:

$$B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt \quad (3.22)$$

where:

$a, b$  are shape parameters,

$B(a, b)$  is the value of the Beta function evaluated at  $a, b$ .

The incomplete-beta function of a variable  $x$  is defined by the following equation [65]:

$$I_x(a, b) \equiv \frac{B_x(a, b)}{B(a, b)} \equiv \frac{1}{B(a, b)} \int_0^x t^{a-1}(1-t)^{b-1} dt \quad (a, b > 0) \quad (3.23)$$

Figure 3.4 shows examples of the incomplete-beta function for various values of the shape parameters  $a$ , and  $b$ . By regression, the appropriate parameters can be obtained that fit the incomplete-beta function to the cumulative relative frequency (CRF) curve obtained from market data. The mathematical property of the incomplete-beta function of a normalized variable  $x$  is that the limiting values are 0 and 1. Thus, the required property of the CDF is satisfied by this function. Also, upon differentiating equation 3.23, we may obtain the PDF resulting from such a CDF. The shape of this PDF is plotted in Figure 3.5. It can be seen that this is a bell shaped curve and can be used as an approximation for a normal distribution. The choice of the parameters results in shifting the mean of the PDF to the right or left.

In addition, it is relatively easy to implement numerical evaluation of the function and its derivative, when compared to other standard probability distributions. Also, the second derivative of the function is a polynomial, and so concavity conditions can be easily established. Thus, the incomplete-beta function is a good candidate for a distribution to fit market data.



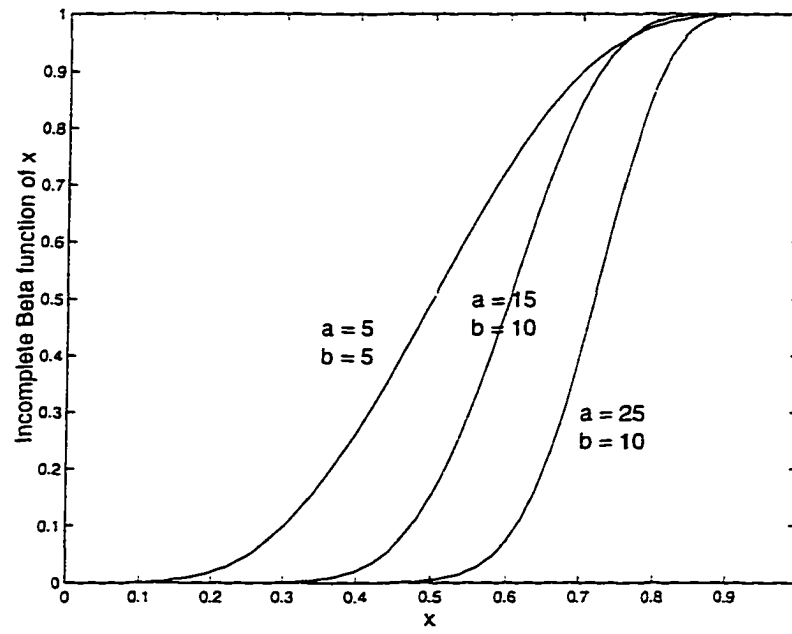


Figure 3.4 Incomplete-beta function.

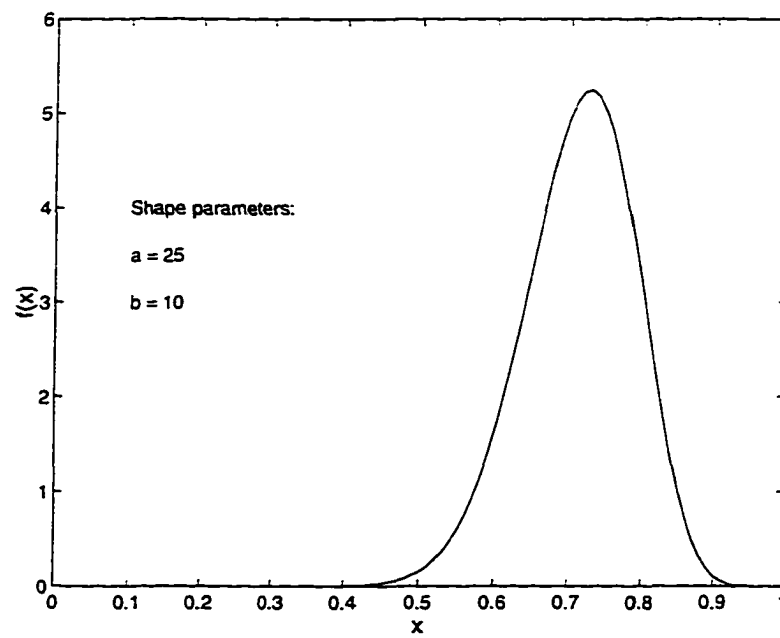


Figure 3.5 PDF from incomplete-beta CDF.

### 3.6 Suboptimal Bidding Using the Incomplete-Beta Function

Now, if we use an incomplete-beta function with parameters  $a$ , and  $b$  to represent the CDF, we need to normalize the variable  $p_b$  so that it can vary between 0 and 1. Let us effect the following normalization:

$$p_b = mx \Rightarrow x = \frac{p_b}{m} \quad (3.24)$$

where  $m$  is the largest unit price in \$/MWH that the participant infers as “sufficient” to virtually *ensure* acceptance. In other words, the participant considers  $m$  to be the farthest outlier of competing competing unit buy bids. Having performed this normalization, let us define a new objective function that is in terms of  $x$  as follows:

$$\begin{array}{cc} \text{Maximize} & \\ x & F(x)(c - mx) \end{array} \quad (3.25)$$

We can now insert the incomplete-beta function in place of  $F(x)$  to arrive at the final objective function.

$$\begin{array}{cc} \text{Maximize} & \\ x & E_{lb}(x) \end{array} \quad (3.26)$$

where  $E_{lb}(x)$  is the lower bound to expected profit and is given by:

$$E_{lb}(x) = (c - mx) \frac{1}{B(a, b)} \int_0^x t^{a-1} (1 - t)^{b-1} dt \quad (3.27)$$

For a seller, a very similar procedure would result in the following  $E_{lb}(x)$ :

$$E_{lb}(x) = (mx - c) \left( 1 - \frac{1}{B(a, b)} \int_0^x t^{a-1} (1 - t)^{b-1} dt \right) \quad (3.28)$$

The normalization procedure is also the same, with  $p_s$  replacing  $p_b$ . The above expressions can be numerically maximized to determine the value of  $x$  (and hence  $p_b$  and  $p_s$ ) that maximizes the lower bound to expected profit. This suboptimal bid is then subject to heuristic tuning along the lines of that presented in [2]. However, before we proceed to maximize the above objective function, it is worthwhile to derive conditions on  $x$

for  $E_{lb}(x)$  to be concave, in terms of the parameters  $a$ ,  $b$ ,  $m$ , and the marginal cost  $c$ . If these conditions lie beyond the constraint boundary for  $x$  ( $x$  lies between 0 and 1), then it is futile to attempt numerical maximization, and we assume that as a fall back, the participant bids the marginal cost. To determine these conditions, we differentiate  $E_{lb}(x)$  twice w.r.t.  $x$  and set the second derivative to be negative.

After performing some simplifications, shown in Section A.2.2, we arrive at the following expression:

$$E''_{lb}(x) < 0 \Rightarrow (a+b)mx^2 - \{(a+1)m + c(a+b-2)\}x + c(a-1) < 0 \quad (3.29)$$

This condition holds if  $x$  lies between the roots of the quadratic expression. This can be easily checked. It can be shown that the concavity conditions for the seller are also identical to the above condition.

If concavity condition holds, then we have a unique maximum over the allowable range for  $x$ . Figure 3.6 shows such a case where a buyer with a marginal generating cost of 12 \$/MWH is considering the bidding decision. The variation of  $E_{lb}(x)$  with  $x$  is shown for a maximum bid price of 14.99 \$/MWH. It can be seen that the lower bound to profit has a maximum of 0.6187 \$/MWH, at  $x = 0.7143$ .

The value obtained for  $x$  is in terms of the normalized bid price. To obtain the suboptimal bid price in \$/MWH, we multiply  $x$  by  $m$ , the normalizing maximum bid price. This results in a suboptimal bid price of 10.7074 \$/MWH with a probability of success of 0.4786. This maximum can be numerically isolated by using any of a number of classical techniques, such as Brent's method, which is a hybrid of quadratic interpolation and golden section search. Source code for the Beta function, incomplete-beta function, and for Brent's method are readily available in the literature, and in this research were obtained from the book *Numerical Recipes in C* [65].

Similar to Table 3.2, suboptimal values can be calculated for the above case of an incomplete-beta function with  $a = 25$ , and  $b = 10$  for various values of  $c$ . These are

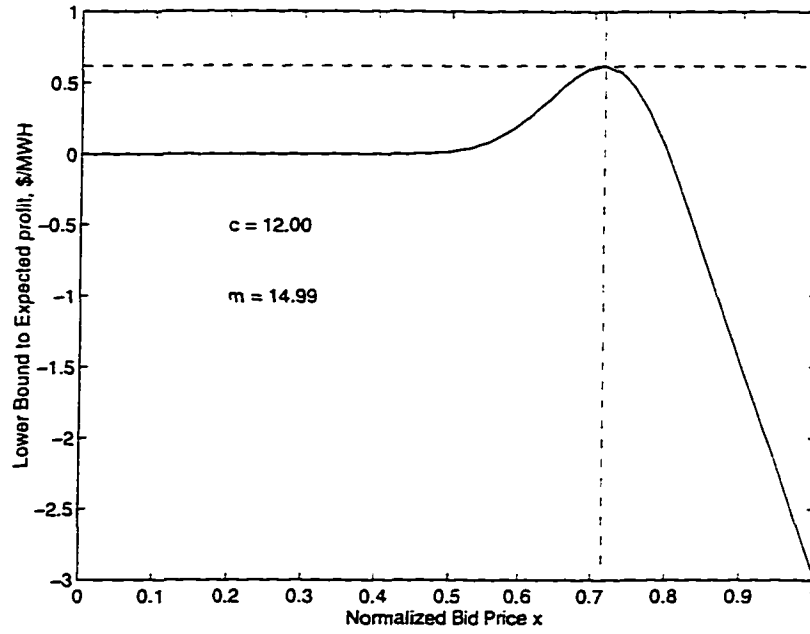


Figure 3.6 Variation of  $E_{lb}(x)$  with  $x$ .

presented in Table 3.3. The values for  $c$  used are once again the same as the values used in the previous two distributions, but the resultant values from optimization are different because of the choice of parameters. Again, it can be observed that the procedure results in suboptimal bids that lead to higher probabilities of acceptance (approaching 1) and higher lower bounds to expected profit (approaching  $c - p_b^*$ ), as  $c$  increases.

Table 3.3 Variation of suboptimal values with marginal cost – beta distribution.

Case	$c$	$x^*$	$p_b^*$	$S(p_b^*)$	$E_{lb}(p_b^*)$
1	8.00	0.5036	7.55	0.0051	0.0023
2	10.00	0.6180	9.26	0.1074	0.0790
3	12.00	0.7108	10.66	0.4607	0.6196
4	14.00	0.7681	11.51	0.7506	1.8664
5	15.00	0.7856	11.78	0.8241	2.6567
6	17.00	0.8085	12.12	0.8998	4.3915
7	17.50	0.8126	12.18	0.9108	4.8442
8	20.00	0.8279	12.41	0.9446	7.1691

### 3.7 Incomplete-Gamma Function Modeling of a Bid Distribution

In this section, the CDF of competing bids is modeled in the form of an incomplete-gamma function. In other words, the competing bids are assumed to be distributed according to a gamma distribution. The gamma distribution has been used to represent many physical phenomena [64] in areas such as failure studies, economics and insurance risk theory. It provides a flexible skewed density over the positive range. The following analysis defines and develops the CDF of the gamma distribution, the incomplete-gamma function, as a possible way to model competing bids.

The expression for the Gamma function is defined as follows [65]:

$$\Gamma(a) = \int_0^{\infty} t^{a-1} e^{-t} dt \quad (3.30)$$

where:

$a$  is a shape parameter,

$\Gamma(a)$  is the value of the Gamma function evaluated at  $a$ .

The incomplete-gamma function of a variable  $x$  is defined by the following equation [65]:

$$P(a, x) \equiv \frac{\gamma(a, x)}{\Gamma(a)} \equiv \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt \quad (a > 0) \quad (3.31)$$

It has the limiting values  $P(a, 0) = 0$  and  $P(a, \infty) = 1$ . Thus the required property of a CDF is also satisfied by this function.

Figure 3.7 shows examples of the incomplete-gamma function for various values of the shape parameter  $a$ . Also, the PDF resulting from the incomplete-gamma CDF can be obtained by differentiating Equation 3.31 and is plotted in Figure 3.8 for a specific value of  $a$ . It can be seen that this PDF is also bell shaped, although the right outlier of the PDF does not occur at a finite value of  $x$ , unlike the incomplete-beta function. This

is because the domain of the incomplete-gamma function is not normalized between 0 and 1. In the context of the strategic bidding application developed in this research, this means that no normalization of the bid-price by a maximum bid-price is necessary.

In addition, similar to the incomplete-beta function, numerical evaluation of the incomplete-gamma function and its derivative is relatively simple. The second derivative is a polynomial, and concavity conditions can be easily established for objective functions such as those described in the previous section. Thus, the incomplete-gamma function is a good candidate for a distribution to model market data. However, because of the use of only a single shape parameter  $a$ , it was observed that a good fit to market data was difficult to obtain when using the incomplete-gamma function. Due to this limitation, simulations supporting incomplete-gamma modeling were not performed. Nevertheless, it is of value to develop a theoretical procedure similar to the incomplete-beta function approach of Section 3.6. Such a procedure could be applied if a satisfactory fit were obtained using an incomplete-gamma function.

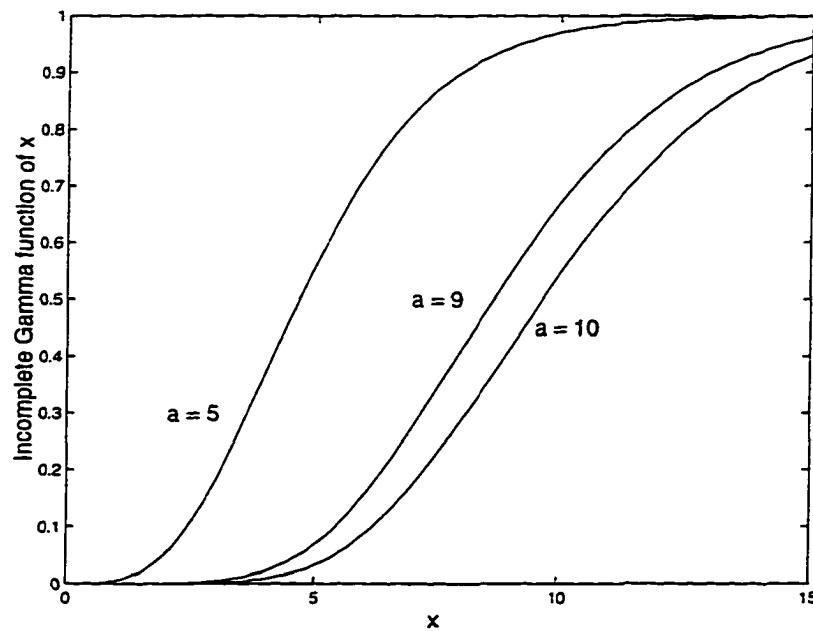


Figure 3.7 Incomplete-gamma function.

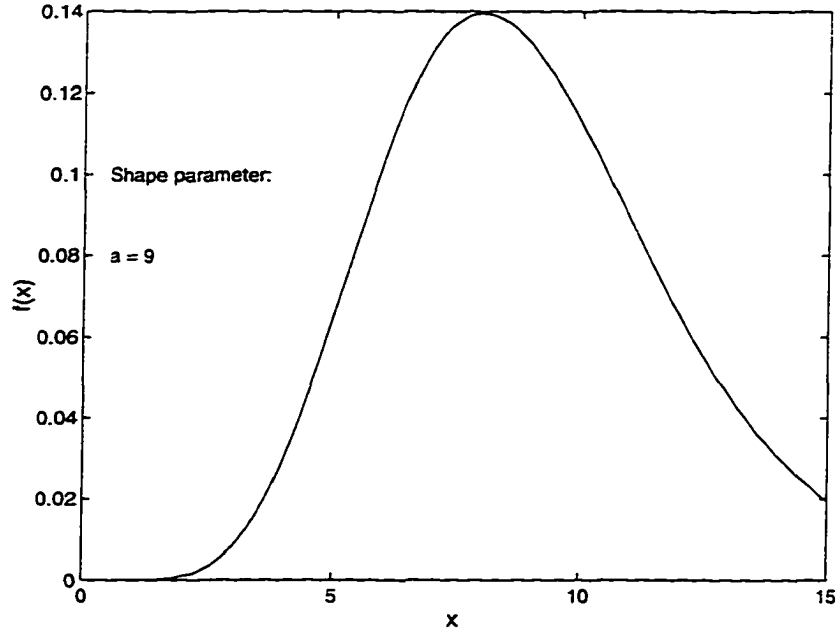


Figure 3.8 PDF from incomplete-gamma CDF.

### 3.8 Suboptimal Bidding Using the Incomplete-Gamma Function

In this section, the theoretical development of a suboptimal strategy by using incomplete-gamma function modeling is presented. Proceeding in a manner similar to that of Section 3.6, we have the following lower bounds to expected profit for a buyer and a seller respectively:

$$E_{lb}(p_b) = (c - p_b) \frac{1}{\Gamma(a)} \int_0^{p_b} e^{-t} t^{a-1} dt \quad (3.32)$$

$$E_{lb}(p_s) = (p_s - c) \left( 1 - \frac{1}{\Gamma(a)} \int_0^{p_s} e^{-t} t^{a-1} dt \right) \quad (3.33)$$

The above expressions can be numerically maximized, with the resulting suboptimal bid being subject to heuristic tuning. Concavity conditions for both buyer and seller, derived in Section A.2.3, result in the following inequation:

$$E''_{lb}(x) < 0 \Rightarrow p^2 - (a + c + 1)p + c(a - 1) < 0 \quad (3.34)$$

where:

$p = p_b$  for a buyer,

$p = p_s$  for a seller.

This condition holds if the bid price lies between the roots of the above quadratic expression. Unlike the case of incomplete-beta modeling, there is no obvious way to check for concavity because the domain of the incomplete-gamma function is unbounded on the positive side. However, participants may perform sanity checks to see if there exists a maximum to the objective function within a certain “allowable” range of bid prices before proceeding with the numerical maximization. If the concavity condition holds, then we have a unique maximum of the objective function. Figure 3.9 shows such a case where a buyer with marginal generating cost of 12 \$/MWH is considering the bidding decision. The variation of  $E_{lb}(p_b)$  with  $p_b$  is shown. It can be seen that the lower bound to profit has a maximum of 1.67 \$/MWH, at  $p_b = 8.5145$  \$/MWH.

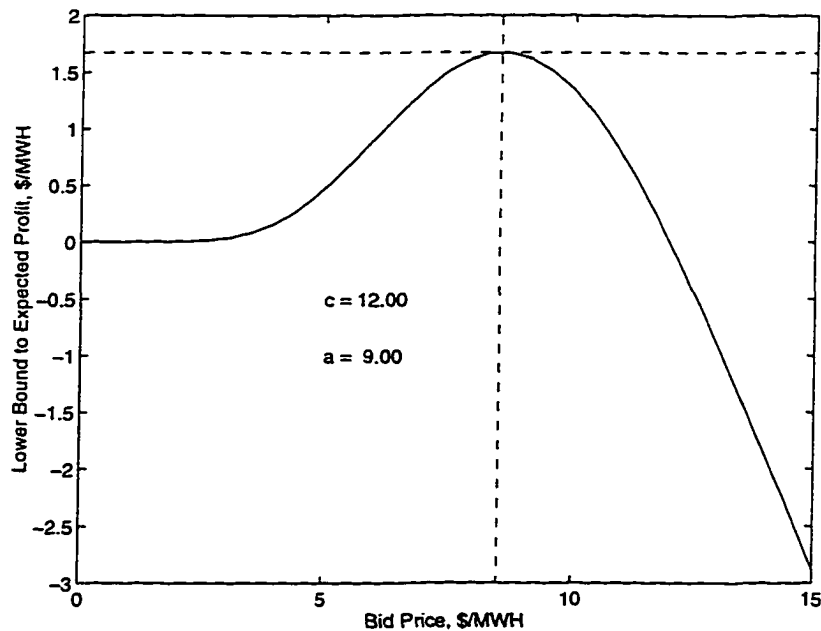


Figure 3.9 Variation of  $E_{lb}(x)$  with  $x$ .



The corresponding probability of success of 0.4789. This maximum can be numerically isolated by Brent's method [65].

Similar to Table 3.3, suboptimal values can be calculated for the above case of an incomplete-gamma function with  $a = 9$ , for various values of  $c$ . These are presented in Table 3.4. It can be observed that the suboptimal bidding procedure results in bids that lead to higher probabilities of acceptance (approaching 1) and higher lower bounds to expected profit (approaching  $c - p_b^*$ ), as  $c$  increases.

Table 3.4 Variation of suboptimal values with marginal cost – gamma distribution.

Case	$c$	$p_b^*$	$S(p_b^*)$	$E_{lb}(p_b^*)$
1	8.00	6.34	0.1897	0.3150
2	10.00	7.52	0.3409	0.8451
3	12.00	8.51	0.4789	1.6692
4	14.00	9.34	0.5881	2.7411
5	15.00	9.70	0.6320	3.3518
6	17.00	10.32	0.7022	4.6896
7	17.50	10.46	0.7167	5.0444
8	20.00	11.08	0.7750	6.9134

### 3.9 Other Candidates for Modeling a Bid Distribution

In selecting the distribution with which to model competitor behavior, this author must admit to having resorted to the trial-and-error algorithm to a certain extent. In general however, a good candidate distribution for modeling competitor bids must have the following properties:

1. The distribution must be continuous, and must have a domain that is bounded on the positive axis, so that only positive bid prices are modeled with a non-zero probability.

2. The shape of the distribution should be flexible by changing the parameters, so that market data can be fitted relatively easily.
3. If a closed form expression for the CDF is not available, then the distribution should be numerically integrable in an efficient manner. This is because a large number of bids might have to be submitted in a relatively short amount of time, and the process should not become computationally cumbersome.

Considering the above factors, several distributions were available to this author. The selection of the beta and the gamma distributions from these was made solely on the basis of source code being readily available to integrate them [65]. However, the following is a list of some other distributions that might be good candidates for modeling competitor bids, provided that a suitable fit can be obtained for the market information. The primary source of information on the distributions described in the following sections is *Probability Distributions* [64], a compact booklet describing various distributions, their properties, and their potential applications.

For all of the following distributions, similar shapes to those presented in Figures 3.4, 3.5, 3.7, and 3.8 can be generated, with an appropriate choice of parameters. Thus, all of the following distributions satisfy the properties of the CDF, as well as resemble the bell shape of the normal distribution.

### 3.9.1 The Pareto Distribution

This distribution has a PDF of the following form:

$$f(x; k, \alpha) = \frac{\alpha k^\alpha}{x^{\alpha+1}} \quad (3.35)$$

where:

$\alpha$  is a parameter,

$x \geq k > 0$ .

The advantages of this distribution are that it is a relatively simple distribution to implement, and inherently bounds the variable  $x$  to be greater than  $k$ . This could be useful in the instances where participants have a good estimate of the lower (or upper) limit of the competing bids. Also, this distribution is widely used for modeling stock price fluctuations, personal incomes and other such empirical phenomena, and could prove to be a good model upon further study.

### 3.9.2 The Exponential Distribution

This distribution has a PDF of the following form:

$$f(x; \lambda) = \lambda e^{-\lambda x} \quad (3.36)$$

where:

$\lambda$  is a parameter,

$x \geq 0$ .

The advantage of this distribution is that it is relatively simple to implement. It is also a special case of the gamma distribution with the shape parameter  $a = 1$ . Its applications are usually in the field of lifetime studies. Applicability to economic phenomena such as the bid modeling area is yet to be studied.

### 3.9.3 The Weibull Distribution

This distribution has a PDF of the following form:

$$f(x; a, b) = abx^{b-1}e^{-ax^b} \quad (3.37)$$

where  $a > 0$  and  $b > 0$  are parameters,

$x > 0$ .

The CDF of this distribution is not easily implemented. However, it provides extra flexibility over the exponential distribution. Its applications have been in the area of

breaking strengths of materials and reliability. Applicability to economic phenomena is yet to be studied.

## 4 BROKERAGE SIMULATOR

In Chapter 3, suboptimal bidding strategies were presented, which incorporate competing bids in the form of distributions, and production costs, to calculate a bid price that maximized the lower bound to expected profit from the bid. In order to test these strategies and to develop additional strategies, we have developed a brokerage simulator, which is described in this chapter. The simulator consists of a bid matching module that performs the function of the broker. Various participants can be simulated by submitting different bid data to the bid matching module. The results from the matching are made available to other modules that calculate the performance of the individual participants based on the results of the bid matching process, which is a simulation of trading activity. Thus, an energy brokerage market is simulated, and from the outputs, the bidding strategies of the participants can be evaluated. Such a simulator could be used by the participants in a real energy brokerage for a variety of functions, such as evaluating their strategies, observing the outcomes of the strategic bidding activity, modeling the behaviors and strategies of key competitors, and training their energy traders.

### 4.1 Rationale for Simulation

The advantages of using a simulator for research purposes are as follows:

- Strategies can be formulated and tested easily in the controlled environment of a simulator.

- Market data for future use can be generated by performing a few rounds of brokerage simulation. Such market data may be difficult to obtain otherwise.
- Market rules and structure can be modified easily in a simulator, thus allowing the extensive study of different strategies under different conditions.
- Simulations help in understanding some of the intricate relationships and effects of various market factors on the effectiveness of strategies.

Thus, there are several advantages in developing a brokerage simulator for research purposes. However, some difficulties exist when participants wish to use such a simulator as a tool to improve their performances in real life situations. These are discussed next.

- Modeling all the rules of the energy brokerage market to the fullest extent is a difficult, time-consuming and ongoing job. Participants might not wish to invest the necessary development time and money required. An option might be to purchase an off-the-shelf product that offers flexibility in modeling various types of energy markets. It remains to be seen how much choice participants would have in such products, and how flexible the products would be. The development and enhancement costs could be distributed among a number of participants, in the case of such products.
- Modeling competitors in the simulator would mean that participants should have some amount of intelligence on the past bidding histories of these competitors. Such intelligence could prove expensive, and its accuracy hard to estimate. An alternative would be to use publicly available transaction price information as a proxy to bid histories, and to modify this information heuristically if necessary. This approach was investigated with limited success in this research and results are reported in the next Chapter 5.

- The success of using past histories to guide future strategies is dependent on future market conditions being similar to those used in the development of the strategy. If this condition is not met, then the bid distributions used in deriving the results of Chapter 3 would be inaccurate, and perhaps render the strategy to be of poor quality. This is a generic problem inherent in many trading strategies in other markets as well. Thus, participants should consider the results from simulations in the light of current market conditions and up-to-date information. Performing simulations for a wide variety of system conditions could be one way of mitigating this problem.

In spite of the above difficulties, using a simulator as a testing and development tool for real energy floor trading does have the same advantages as those listed for research purposes. In addition, such a simulator can also be used by participants for training purposes. A less likely, but possible use for such simulators would be by regulators to test various market structures and their effects on electricity prices, market power of individual participants, etc.

## 4.2 Simulator Overview

Figure 4.1 shows a functional overview of the different modules that constitute the brokerage market simulations performed to test the strategies developed in Chapter 3. Most of the modules shown in the figure were developed and implemented as part of this research. The main programming environment was C/C++ on a HP-9000 C110 Workstation. Some of the modules were also developed as Matlab v.4.0 m-files. The different modules interface through text input/output files, and are invoked and managed by Perl scripts. The use of Perl enhances the flexibility of the simulator. The free nature of Perl also adds to portability to different platforms. As nearly as possible, programming language usage conforms to ANSI standards. The compilers used were Gnu's *gcc* and

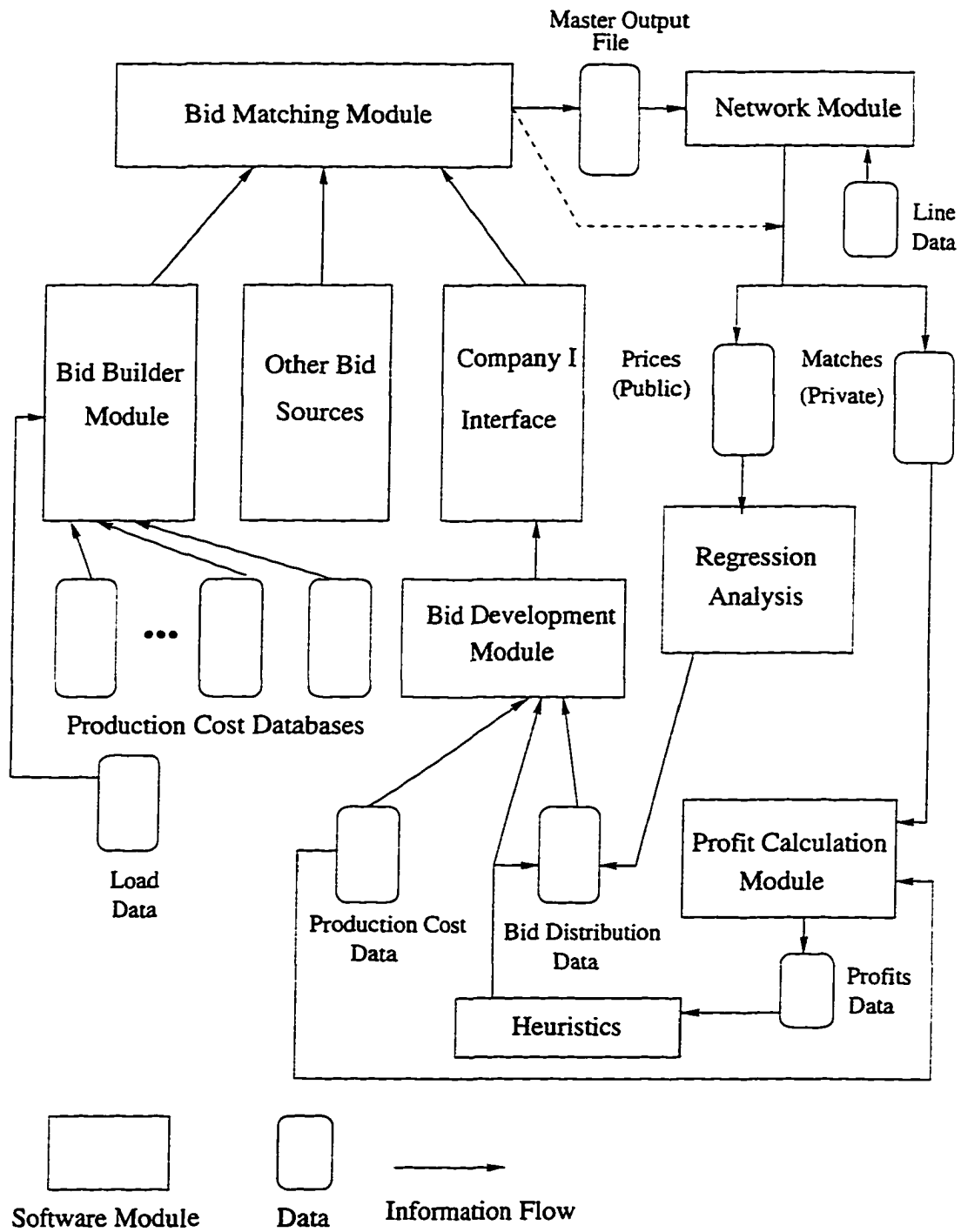


Figure 4.1 Brokerage simulator – functional overview.



*g++*. This further enhances portability to various UNIX platforms. Porting to a PC platform has not been examined at this point.

#### 4.2.1 Bid Matching Module

The bid matching module performs the function of the broker in an energy brokerage. The objective of the bid matching module is to match buy and sell bids in a manner such that the net savings (the difference between buy bid and sell bid) is maximized. In the simulator, it is implemented as a program written in C++. The inputs to the module are hourly bids to buy or sell energy, at a given price. The bids are in fixed block sizes (of say, 20 MW). The algorithm for bid matching is the high-low matching algorithm, commonly found in the literature for fast and simple implementations. In this algorithm, the buy bids are sorted in descending order, the sell bids are sorted in ascending order, and the highest buy bid is matched to the lowest sell bid, and so on. Matching stops when the current buy bid under consideration is less than the corresponding sell bid, i.e., no further savings are realized by matching. The bids that are not matched are considered rejected. This algorithm is equivalent to an LP solution in the absence of transmission constraints. In the presence of transmission, the solution is suboptimal. The following are the inputs to the bid matching module:

1. Bids from company I interface
2. Bids from bid builder module
3. Bids from other bid sources

The following is an example of a bid file. As long as the information shown in the bid file is present in the order shown, the bid matching module does not discriminate between bids from any of the above three sources.

Company\_Name(Max 20 chars)

-----

cat

Beg\_Hr End\_hr Split/All #Blocks Qty Price Type

-----

1	1	Split	1	20	8.99	Buy
1	1	Split	1	20	8.83	Buy
:	:	:	:	:	:	:
:	:	:	:	:	:	:
1	1	Split	1	20	9.09	Sell
1	1	Split	1	20	9.15	Sell
:	:	:	:	:	:	:
:	:	:	:	:	:	:

The following are the outputs from the bid matching module:

- If transmission is not considered, the bid matching module produces the final match list. In this case, the outputs are:

1. *Individual match information files.*

One file is written for each participant. It contains all the matches that the participant was involved in, either as a buyer or as a seller, in each hour during the bidding period specified. The information contained includes buyer name, seller name, hour number, quantity bought/sold, and the transaction price. The transaction price is set at the average of the buy and the sell bids. This file is considered "private". In other words, when the simulator is upgraded for implementation on a relational database, this file can be accessed only by the concerned participant.

2. *Price and volume information files.*

One file is written for each hour in the bidding period. It contains the hour,

quantity and transaction price of every match for that hour. No information is displayed regarding the buyer or seller. This information is considered “public”, and may be accessed by all participants.

### 3. *Master output file for transmission evaluation.*

Even if transmission option is turned off, a file is produced containing all the information present in the individual match information files, for use by the transmission evaluation module if need be. A sample master output file is shown below. The information includes seller, buyer, hour number, block size in MW, transaction price, buy bid, and sell bid.

cat	Sell to:	wings	1	20	9.04	16.62	12.83
cat	Sell to:	swamp	1	20	9.09	15.22	12.155
cat	Sell to:	swamp	1	20	9.15	14.58	11.865
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:

### 4. *Output listing of the matching process*

A list file is produced containing number of bids submitted by each participant, number of bids matched, and all transaction information, including any error messages. The purpose of this file is for debugging purposes, but the information can be used for purposes of evaluating market structure and performance.

- If transmission is considered, the bid matching module only produces the master output file for transmission evaluation, and the output listing of the matching process.

## 4.2.2 Transmission Network Module

The transmission network modeling included in the brokerage simulator is of rudimentary detail. This is because, at the time of developing this simulator, it is still not

clear what the consensus model is going to be in the power industry for modeling transmission pricing, available transmission capacity, loss allocation, etc. Thus, a very simple DC-power flow capability has been added with the view that when such a consensus model evolves in the market, this transmission network model can be upgraded to reflect current models. Currently, the module is implemented as a C++ program that can be invoked by the user if desired. Currently, the network module uses the master output data described above, and evaluates each match to detect thermal line flow and economics violations. The following assumptions are made:

1. Each participant has a preassigned bus in a fixed system network, which is the designated power injection bus for that participant. All transactions that result from bids are modeled as positive (negative) injections of the bid quantity at the seller's (buyer's) designated bus.
2. Only thermal line limits are considered in the model.
3. Matches (proposed transactions) from the bid matching module are considered in decreasing order of the magnitude of the resultant savings. If a match results in a violation of thermal limits, then it is rejected.
4. The thermal limits are obtained from an input file. Under the current model, the thermal limits for all periods are assumed to be the same, and are assumed to include any previously scheduled transactions *excluding* those accepted by the broker through the bidding process. These transactions include any bilateral transactions entered into by participants.
5. Transmission pricing is performed by the MW-mile method, wherein the resulting transmission charge is determined as the change in net MW-mile costs caused by the transaction. The net MW-mile costs are calculated as the sum of the product of the absolute value of line flows, the corresponding line's MW-mile tariff, and

its length in miles. The tariffs and the line lengths are assumed to be known in advance to the network module. Each line has its own unique MW-mile tariff. In this model, only one tariff is assumed for all times. However, it is a relatively simple task to provide the capability of multiple tariffs, one for each period, such as on-peak, off-peak, etc.

6. If the transmission charge for a match is more than the savings resulting from the match, the match is rejected.
7. Transmission charges from the final set of accepted matches are equally divided between the seller and the buyer.
8. The assumption made regarding transmission charge allocation is that all fees are initially paid to the broker or some other entity, who disburses the charges to the appropriate owners. In other words, transmission charge allocation is not dealt with directly in the simulator.
9. Transaction fees payable to the broker are not currently modeled in the simulator.

While the above assumptions are admittedly very limiting to the capabilities of the network module, they have been made to simplify the implementation. If a more detailed network module were available to the participants, there is theoretically no reason why the current network module cannot be replaced by the more detailed module.

The inputs to the network module are:

1. The master output file from the bid matching module.
2. Network data files with line reactance, line length, and MW-mile tariff information.

The outputs from the network module are:

1. Match information files that are written for each participant with details of transactions that the participant was involved in. The following sample is an example

of the match information written by the network module. It contains information regarding seller, buyer, hour, quantity, transaction price and transmission cost. For a seller, the price information represents revenue, and transmission cost information represents expenditure. For a buyer, both represent expenditure.

			Hour	Qty	Price	TransCost
cat	Sell to:	wings	1	20	12.83	5.35
cat	Sell to:	swamp	1	20	12.155	3.775
cat	Sell to:	farm	1	20	9.515	5.14167
:	:	:	:	:	:	:
:	:	:	:	:	:	:

2. Price information files for each hour containing limited details of all the matches that are approved for that hour. The following sample is from a price information file. If transmission is not modeled, then this file is written by the bid matching module, without the transmission price information. This data is considered public and may be accessed by all participants.

Hour	Quantity	Price	Transmission
1	20	12.83	5.35
1	20	12.155	3.775
1	20	11.865	3.79167
:	:	:	:
:	:	:	:

3. Accepted transaction files that are considered confidential and are not accessible by anyone except the broker or the independent transmission system operator (ISO) for evaluation purposes. The following is a sample from the accepted transactions file.

cat Sell wings 1 20 9.04 16.62 12.83

Energy Cost Savings = 151.6

Transmission Costs = 5.35

```
cat Sell swamp 1 20 9.09 15.22 12.155
```

```
Energy Cost Savings = 122.6
```

```
Transmission Costs = 3.775
```

```
:
```

```
:
```

4. Rejected transaction files which are considered confidential and are not accessible by anyone except the broker or the independent transmission system operator (ISO) for evaluation purposes. The following is a sample from the rejected transactions file.

```
cat Sell zoo 1 20 9.42 9.49 9.455 (Economics Don't Justify TransCosts)
```

```
cat Sell zoo 1 20 9.45 9.47 9.46 (Economics Don't Justify TransCosts)
```

```
:
```

```
:
```

5. As part of the output from the network module, the final line flows after the last accepted match for each hour are reported. The following is a sample from this data.

```
Line Flow Data at the end of Hour 1
```

```
Flow from bus 0 to bus 1 is -94.6172
```

```
Flow from bus 0 to bus 9 is 156.422
```

```
Flow from bus 1 to bus 0 is 94.6172
```

```
Flow from bus 1 to bus 2 is 187.112
```

```
Flow from bus 2 to bus 1 is -187.112
```

```
Flow from bus 2 to bus 3 is 125.092
```

```
Flow from bus 2 to bus 25 is 39.0101
```

```
:
```

```
:
```

### 4.2.3 Bid Builder Module

This module produces the competing participants' bids from production cost databases. These production cost databases are obtained from off-line unit commitments performed on competing companies' generating unit mixes. If the participants do not have an accurate idea of the competing companies' generating unit characteristics, the production cost databases can be obtained from the best estimates that the participants can make about the mixes. Since the unit commitment program used in this research is written in Matlab, the bid builder module is also implemented as a Matlab program. The inputs to this program are Matlab variables obtained from the base case unit commitment runs. Both buy and sell bids are obtained by performing repeated economic dispatches based on the base case unit commitment. The maximum energy that can be bought and sold in a given hour is determined from the minimum and maximum on-line generating capacities from the base case, and a fixed spinning reserve percentage input by the user. The output is the bid file in the format shown earlier in this section.

### 4.2.4 Other Bid Sources

This module is used when the participant wishes to model some other type of competitor, whose behavior cannot be modeled accurately using the production cost model. Examples include power marketers, load aggregators, etc. So long as the other bid sources are capable of giving an output in the format shown for bids, the bid matching module will be able to accept the inputs. Currently, this feature is not used in the simulator, but adding this feature involves minimal work.

### 4.2.5 Bid Development Module

This module uses the bid distribution data, load data, production cost data and heuristics, and calculates the suboptimal bids for each block of energy that the pro-



duction cost data file contains. The algorithms for finding the suboptimal bids are implemented as C++ programs invoking C functions. The following sections briefly describe the two kinds of bid development algorithms implemented: polynomial function based and incomplete-beta function based.

#### 4.2.5.1 Polynomial Modeling

Suboptimal bids are determined as roots of Equations 3.17 and 3.20 for buy and sell bids respectively. Polynomial root finding is performed by Laguerre's method. Implementation is by slightly modifying the functions *laguer* and *zroots* found in [65], and calling these functions from the bid development function repeatedly. Laguerre's method does return both real and complex roots for the polynomial, so the complex roots are eliminated in the modified version. Furthermore, the suboptimal bid is chosen by calculating the lower bound to expected value of profit for each of the positive real roots, and picking the root that yields the maximum lower bound as the solution. Also, sanity checks are implemented where the suboptimal buy bids do not exceed the decremental cost, and the suboptimal sell bids are greater than or equal to the incremental cost.

#### 4.2.5.2 Incomplete-beta Modeling

Suboptimal bids are determined by numerical maximization of the objective function given by Equations 3.27 and 3.28. The search range for the normalized suboptimal bid price is  $[0, 1]$ , by definition of the beta function<sup>1</sup>. Before the numerical maximization is performed, concavity conditions are tested by verifying that concavity conditions given by Equation 3.29 do not result in a range that is outside the allowable range for the normalized suboptimal bid price. If this is not the case, then numerical maximization will not result in a usable bid price, so the C++ program written for the incomplete-

---

<sup>1</sup>There is a further reduction in the acceptable search range, which results from the sanity checks mentioned in the previous section. Thus the search range is  $[0, \frac{c}{m}]$  for a buy bid and  $[\frac{c}{m}, 1]$  for a sell bid.

beta model defaults to marginal cost for a bid price, in the case when the concavity conditions are not met. If concavity conditions are not violated in the search range, then the numerical maximization is performed by using Brent's method for minimization, as presented in [65]. Brent's method in one dimension is a combination of parabolic interpolation and golden section search. It is a very fast and robust way to search for a local minimum in a given range, provided the range brackets such a local minimum. To use this method, the objective function to be maximized is multiplied by -1 before applying Brent's method. The specific C functions used from the book *Numerical Recipes in C* [65] are *brent* for minimization, *betai*, *betacf* and *gammln* for calculation of the incomplete-beta integral.

In addition, lower bounds to the probability of acceptance, and/or to the expected profit, can be specified manually by the user. If the suboptimal solution does not satisfy these lower bounds, then the program defaults the bid price to equal marginal cost. This was done to model the fact that participants need not always "trust" the suboptimal bidding algorithm to come up with a suitable bid price.

#### 4.2.6 Profit Calculation Module

The profit calculation module is a program written in C++. It takes as inputs: the match files written by the network module (or the bid matching module if the transmission network module is not included), and the production cost data. Then it calculates the profits made by the participant for each hour. The net energy bought or sold by the participant, and the corresponding net revenue or expenditure, is first calculated by processing the match information. Then the production cost for the corresponding net sale/purchase blocks is determined from the production cost data. The difference between the two is the profit for that hour. The module then calculates the total profits for the bidding period by adding the profits from all hours. The output is a profit file. A sample of the profit file is shown below. It contains information on net number of blocks bought or sold, net expenditure, and net savings.

## PROFIT INFORMATION

Hour	Blocks	Type	SavedCost	AmtPaid	Savings
1	-10	1	-1843.40	-2092.90	249.500
2	-8	1	-1476.00	-1505.80	29.8000
3	-9	1	-1671.40	-1759.30	87.9000
4	-12	1	-2239.40	-2502.00	262.600
:	:	:	:	:	:
:	:	:	:	:	:
Total					18930.0

#### 4.2.7 Regression Analysis

For polynomial modeling, regression analysis using Matlab provides estimates of the parameters of the competing bid distributions using the price data output by the network module (or the bid matching module). The data from the prices output files are first collated into appropriate categories, such as on-peak or off-peak hour prices by using Perl scripts. The user can select which hours are on-peak and which hours are off-peak. Then the resulting collated price files are sorted, and input to a Matlab function that constructs a cumulative relative frequency (CRF) histogram of the prices. This histogram is used as an approximation of the CDF of buy/sell bids. The user can select the number of bins in the histogram (which affects the accuracy of the curve fit). Then, the Matlab function *polyfit* is used to find a least-squares polynomial fit of a user specified degree, to the relative frequency histogram.

For incomplete-beta modeling, non-linear regression is required to fit an incomplete-beta function to the CRF histogram. Such a function is available in the SPSS package. In this package, two options exist to perform the regression. One is the Levenberg-

Marquardt algorithm and the second is the sequential programming algorithm. For the data used in this research, it was found that the sequential programming algorithm resulted in a better convergence to the parameter estimates.

From the above two short descriptions, it can be seen that the regression analysis feature of this simulator is primarily obtained from external packages. If the participant desires, they could develop custom applications to suit their modeling needs. In this research however, the focus is on how to use the results from fitting distributions to simulator-generated market data, rather than on the fitting itself. So, the regression analysis implementation is of a rudimentary nature.

#### **4.2.8 Heuristics**

The distribution parameters obtained from the previous section use transaction prices as proxies to actual bid data, which are confidential. But based on the performance of the participant's strategies in initial rounds of bidding, it is possible that the participant has obtained some knowledge about the actual bid distribution. This market "wisdom" has been aggregated and is functionally represented by the heuristics module in Figure 4.1. The heuristics may serve to modify both the distribution parameters, as well as the bids themselves. The first kind of heuristics affects bids indirectly, and the second kind, directly.

## 5 SIMULATION RESULTS AND ANALYSIS

In this chapter, the results of the simulations performed using the brokerage simulator described in the previous chapter for various energy brokerage market scenarios are presented and analyzed. The simulations and the analyses presented are not proofs that the strategies developed here are correct, or that they work under all circumstances. In fact, they will not. Further, the scenarios presented are only a small subset of the complex possibilities that exist even in a market with as simple a structure as the one assumed in this dissertation. The goal behind the simulations is primarily to examine some of the mechanisms by which the strategic bidding theory developed in Chapter 3 can be implemented with limited information, modified heuristically to compensate for the limited information, and tested. We cannot conclusively say that the strategies improve bidding performances under most scenarios until extensive simulations are performed for different market conditions. However, the simulations performed to date indicate that the bidding strategies theoretically developed can be implemented relatively easily, and show some promise of improving performances of participants.

### 5.1 Simulator Use Overview

The simulator itself is presented as a tool an individual participant could use to test their strategies before using them in actual bidding situations. This testing could proceed along the following steps:

1. Generate off-line unit commitment and production costing simulations on own generating system.
2. Prepare production cost data for competitors if detailed modeling data is available, or use “generic” numbers to generate competitor cost information.
3. Use the bid-builder module to generate bids for own system, and competitors’ systems, from the cost data and submit these bids to the bid-matching module.
4. Perform bid-matching and profit calculation for each participant for the given bidding period. This is the “base-case” simulation, where all players bid their marginal costs.
5. Use the transaction price data from the base-case simulation as public information, and generate approximations to bid distributions, using the regression module<sup>1</sup>.
6. Use the bid distribution parameters to generate suboptimal bids for user’s own system, by using one of the strategies developed in previous chapters. If need be, apply heuristic tuning to these bids (described later in this chapter) to compensate for the errors in approximating bid distributions from transaction price distributions.
7. Submit the suboptimal bids for user’s own system to the bid matching module, while keeping all other bids the same.
8. Evaluate profits from this new simulation against the base-case profits, to determine the effectiveness of the strategy.
9. Translate any insights obtained into knowledge that can be used in the future.
10. Repeat the procedure for various scenarios.

---

<sup>1</sup>At present, regression is performed manually, by invoking SPSS or MATLAB. However, a future enhancement of this simulator should include an automated module to perform regression “on-the-fly”.

In the following sections, the simulations described were performed by using the simulator in a manner similar to that just described.

Thus far in this chapter, we have focused on evaluating the *profits* made by the individual participants. However, the main motivation behind the regulatory changes occurring in the power industry seems to be reducing the *prices* that the rate-payers (or the customers) pay the producers. As of yet, it is unclear how the power companies propose to share the profits they might stand to make in the bulk power market with the end users. Indeed, it is not clear if a net decrease in electric rates would occur at all. Therefore, we make the distinction in this research, between improving the performance of the individual companies, and reducing electricity rates: the strategies developed and tested in this research focus on improving the profits of the individual companies that employ them. No effect is presumed or predicted on the price of electricity to the end user.

## 5.2 Overview of the Test System

This section briefly describes the test system used to run the simulations. The test system used in the simulations consisted of 8 companies, which are all assumed to be participants in the brokerage market. The eight systems are assumed to be utility-like entities, in that they all own generating resources, and have to satisfy a native load. This is not a requirement, but was chosen for convenience. Further, it is assumed that the generating capacity of each participant is sufficient to satisfy its corresponding generating requirements, including a spinning reserve requirement of 15% of the hourly forecast load. This is verified for each system by performing a priority list based unit commitment<sup>2</sup>. The companies will be denoted by using numbers, as company 1 through company 8. Table 5.1 shows the company summary information, including number

---

<sup>2</sup>The unit commitment program used is based on a priority list based program developed by Sridhar Kondragunta and enhanced by this author. It is implemented in Matlab.

of units, peak load, sum of maximum and minimum capacities assuming all units are online<sup>3</sup>.

The relevance of the native load requirement is justified in the current transitional environment where traditional utilities are preparing for competitive markets, while still having to satisfy native load commitments. In the future, less regulated environments, the native load requirements will be replaced by firm generation contracts that are already in existence at the time the bidding decision is being made. Thus, the strategies developed in this research with the assumption of an underlying “base-case” unit commitment are still expected to be relevant and applicable.

Table 5.1 Test system data – company summary information.

Company	Units	Peak Load (MW)	$\sum P_{min}$ (MW)	$\sum P_{max}$ (MW)
1	16	2,050	770	2,940
2	4	150	62	224
3	14	1,775	1,018	2,523
4	4	175	85	247
5	22	5,025	3,150	7,182
6	4	225	93	310
7	4	100	50	134
8	16	1,975	948	2,804
Totals	84	11,475	6,176	16,364

All participants are assumed to be bidding to buy and sell simultaneously in the brokerage market. This assumption is made so that no preconceived notions exist about the nature of the participants.

In the first part of this chapter, the scenarios presented are simulations of the effects of strategies on the bidding performances of the participants, *without* modeling transmission. In the latter part, transmission is also included to a limited extent in the modeling. For these scenarios, the transmission system underlying the brokerage market is assumed to be the IEEE 30 bus reliability test system (IEEE-RTS), which has been

---

<sup>3</sup>The assumption of all units being online is made only for presenting the data in the table, and not for the simulations.



slightly modified. The modifications are:

- The slack bus has been renumbered to be bus 0, instead of bus 30.
- The generators in the slack bus and bus 20 have been dropped. The slack bus generator has been dropped because the model used to calculate line flows is a DC power flow model, and so losses are not considered. The generator on bus 20 has been dropped so that the total number of generators can be limited to 8, the number of participants in the system. Bus 20 was chosen because bus 19, in close proximity, is the designated bus of one of the companies. The resultant modified IEEE-RTS system is shown in Figure 5.1.
- Only Y-bus data is used from the IEEE-RTS test system. All the generators shown in Figure 5.1 represent the entire generating system of each participant.

The modifications were made for simplifying the transmission modeling, and for keeping the number of generating buses in the system to be equal to the number of participants.

Further, for pricing purposes, all lines are assumed to have a length of 50 miles, and a MW-mile tariff of 0.01 \$/MW-mile. These numbers were assumed because no economic data was available for the IEEE-RTS system. The magnitudes of the numbers used were chosen based upon trial and error simulations, such that the transmission price component of the transactions were approximately in the 4 \$/MWH range. This was within the range of currently posted transmission usage tariffs<sup>4</sup>.

### 5.3 Initial Calculations and Simulations

In order to develop bid distributions from past bidding history, and to use these distributions to develop bids for the market, we needed some starting point for the

---

<sup>4</sup>For example, Pennsylvania Power & Light tariffs range from 0.05 to 0.2 \$/KW per day for reserving capacity, which translates to approximately 2 to 8 \$/MWH.

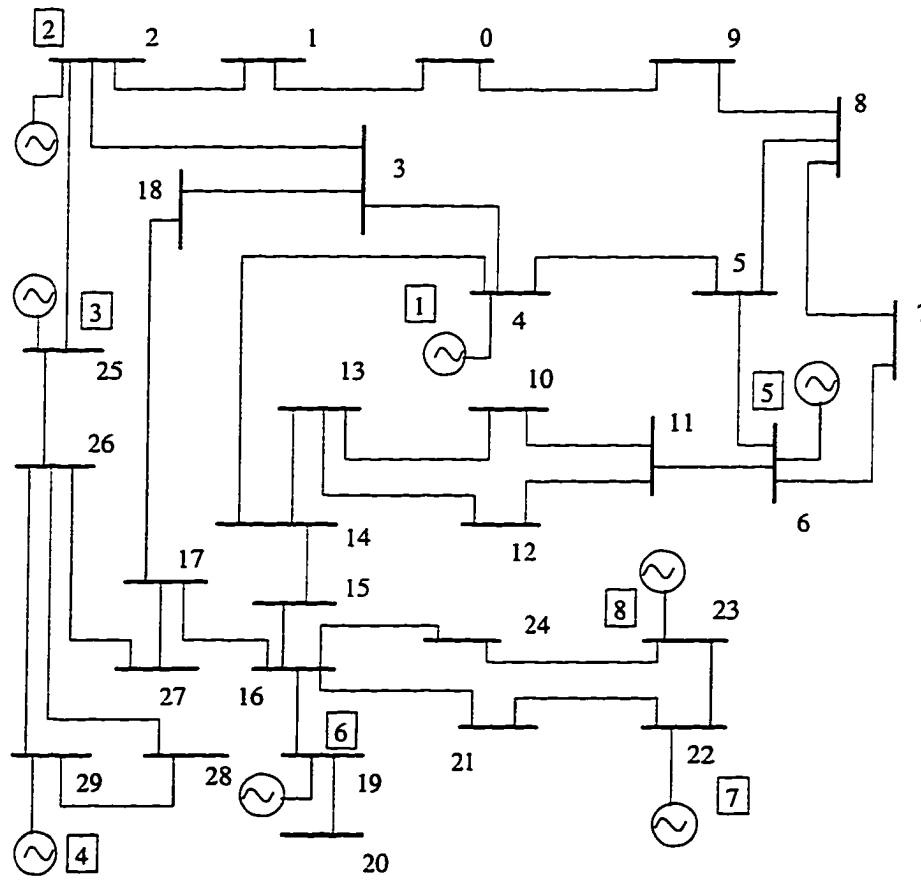


Figure 5.1 Modified IEEE-RTS system

strategies to use. Also, to consistently compare the performance of a particular strategy for different participants, we needed a benchmark. In the following sections, such a “base case” was obtained by assuming that all the participants of the market operate under perfect competition assumptions. Thus, the bids submitted to the broker by the participants is simply their marginal cost/value of energy. This cost is obtained by using the Matlab-based bid-builder module, outlined in Section 4.2.3. This module uses a one-week unit commitment as a basis for generating block-incremental and block-decremental production costs for each hour.

Another possible approach for obtaining a benchmark is to perform a combined unit commitment of all the companies’ units, serving the combined loads of the all companies. This would have resulted in the traditional pool-dispatch of the units, and provided a benchmark that considers the case that maximizes system savings, and reduces the system costs. However, we opted for the competition assumption, with individual unit commitments, for this research.

The number of blocks bid for in each hour was determined by the minimum and maximum online capacity for that hour as scheduled by the unit commitment run, the native load for that hour, and the reserve requirements. The block-incremental/block-decremental costs were obtained by running economic dispatches with and without the block being sold/bought, and using the difference in costs to cost the block. The resulting bids were used as a base-case bidding strategy for each participant.

The base-case bids developed by the above module were processed by the bid-matching software described in Section 4.2.1. The resulting transaction prices for each hour were considered to be the publicly available information to the participants. Based on this information, the participants were assumed to develop suboptimal bidding strategies, using the results developed in this dissertation thus far. These bids were then submitted to the bid-matching module to evaluate the effectiveness of the particular strategy.

Table 5.2 shows the resultant profits of each of the companies from bidding according to various strategies, for a period of 168 hours. In the second column, labeled "INIT", the results shown are the profits from bidding marginal costs, in other words, this column represents the base case described in Section 5.3. In submitting the bids to the bid-matching module, it was assumed that all the participants were bidding simultaneously to sell and buy energy.

Table 5.2 Polynomial model – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	19,649	20,233	19,229	18,686	19,394	19,609	18,943	18,943	19,299
2	19,212	19,244	21,505	19,213	19,243	19,212	19,321	19,253	19,212
3	13,714	13,489	13,646	12,435	13,729	13,721	13,517	13,691	16,159
4	8,118	8,118	8,135	8,118	9,166	8,117	8,113	8,118	8,119
5	83,407	82,889	81,391	79,032	81,759	83,419	79,909	82,484	77,504
6	36,440	36,441	36,609	36,441	36,441	36,441	40,906	36,506	36,441
7	11,784	11,784	11,795	11,784	11,784	11,784	11,826	13,033	11,784
8	17,644	17,468	17,524	17,524	17,596	17,648	17,428	17,583	16,916
Tot.	209,969	209,730	209,835	205,966	209,112	209,953	209,967	209,969	204,577

An examination of the match information after the fact, however, showed that companies 2 and 5 were predominantly selected by the module to be sellers, while the other companies were primarily buyers, in most of the hours.

## 5.4 Polynomial Model

In this section, simulations from using the polynomial modeling results developed in Section 3.4 are presented. Using the transaction prices data generated from the base-case simulations above, a cumulative relative frequency (CRF) curve of the prices was obtained. The prices were divided into on-peak and off-peak prices, using Perl scripts to collate the data into appropriate hours. In these simulations, hours 8 through 22 were assumed to be on-peak for each day, with the other hours being classified as off-peak.

This was done following current power industry convention<sup>5</sup>. Then, a MATLAB script was used to curve-fit a 20th degree polynomial to the on-peak and off-peak CRF curves. Thus, two different polynomials were used to model on-peak and off-peak periods. These polynomials are approximations to the CDF of transaction prices occurring in the market. In the absence of any information on the actual *bids* that resulted in these prices, this research attempts to use the CDF as proxies for the bid distributions themselves. The resulting bids then can be subjected to heuristics, which are attempts to guess more closely, the actual bid distributions.

As an initial step, bids were developed that do not involve any heuristics. In other words, the coefficients of the polynomial obtained are directly assumed to be the coefficients of Equation 3.14. Following this, Equation 3.17 was solved for each bid block, using the associated generating cost determined by the bid-builder module, and the coefficients of the polynomial. Thus, for each block that the participant bids, a suboptimal solution was obtained from the results developed in Section 3.4. The calculations were performed by a C++ program on a HP C110 workstation, and the approximate execution time for the largest company, company 5, was 192 CPU seconds. This execution time was for suboptimal bid calculations for the 168 hour period, and involved the pricing of 18,429 blocks. The execution time for the other, smaller companies was substantially lower. Thus, the required calculations can be performed very efficiently in a relatively short period of time.

Columns 3 through 11 in Table 5.2, labeled “1-SUB”, “2-SUB”, etc., represent profits from bidding scenarios where each company was in turn assumed to use the suboptimal strategy, while the other companies were assumed to submit the same bids as in the base case, i.e., the marginal cost based bids. Thus, the column labeled “1-SUB” represents the case where only company 1 was assumed to be bidding suboptimally. To evaluate

---

<sup>5</sup>Some utilities use hours 7 through 22 as on-peak, while others use “shoulder” hours between on- and off-peak hours.

the effectiveness of the strategy, we compare the profits in row 1 of this column to the corresponding profits in row 1 of the column labeled "INIT". From this comparison, we can see that there is an increase in the profits of company 1 when compared to the base case. The other numbers in the column labeled "1-SUB" represent the profits of the other companies for this scenario. Table 5.3 summarizes the results from these simulations. In this table, the column labeled "INIT" is identical to the column in Table 5.2 of the same label. The third column, labeled "SUBOPT" contains the profits from the scenarios when the company corresponding to the row label, uses the suboptimal strategy. In other words, this column shows the diagonal elements of the section of Table 5.2, from columns 3 through 11.

Table 5.3 Polynomial model -  
suboptimal bidding  
profits in \$.

<i>Company</i>	<i>Scenario</i>	
	INIT	SUBOPT
1	19,649	20,233
2	19,212	21,505
3	13,714	12,435
4	8,118	9,166
5	83,407	83,419
6	36,440	40,906
7	11,784	13,033
8	17,644	16,916

Comparing the numbers in each row of this column with the corresponding row of column 2 of the same table, we can see the effect of the strategy on the bidding profits of the company. It can be seen that with the exception of companies 4 and 8, the strategy resulted in an increase in bidding profits for all the other companies.

Upon examination of the bids, it was observed that all the suboptimal sell bids were greater than or equal to the base-case bids, and all the suboptimal buy bids were less than or equal to the base case. This implies that the suboptimal bid development

algorithm works correctly, by determining a mark-up/mark-down on the marginal cost for the selling/buying decisions. Also, upon examination of the matches, it was observed that for all the companies, the number of bids matched was lower than in the base case. This is also not surprising, since the bid-matching module only matches a pair of buy and sell bids if a positive savings results. Adding mark-ups/mark-downs would move the supply and demand curves closer to each other, thus resulting in lower volume.

The structure of the market is such that, if this mark-up/mark-down is too high/low, then the bid stands the risk of being rejected in favor of competing bids. Now, the competing bids are represented by an approximation to their probability distribution, and so the negative effects of the strategy can be explained by the fact that the distributions do not represent the competing bids well for that particular participant. The fact that the same strategy, using the same distribution, resulted in an increase in profits for 6 of the 8 companies can be explained by one of the following two reasonings:

- Errors in the assumed distribution of competing bids were such that the resulting suboptimal bids were shifted in the direction that increased probability of acceptance, so that, while bidding profits were reduced, bid acceptances were not.
- Errors in the assumed distribution did result in increased bid rejections in favor of competing bids, but the production cost structure of the participant was such that, in spite of this effect, there was a net increase in profits.

Because of the large amount of data generated by the simulator, it is hard to understand clearly which effect predominates for each participant. However, the results indicate that the suboptimal strategy does result in some improvements of profits even for the rather simplistic modeling used here. It would be of interest to see the effect of detailed past-history modeling, perhaps using a distribution for each hour in the simulation period of 168 hours, on the effectiveness of the strategy. But such a study is beyond the scope of the time and resources available to this author.

However, the following attempt was made to implement a heuristic modification of the suboptimal bid. The main reasoning behind this heuristic is that, since the suboptimal bids resulted in mixed results, both an increase and a decrease in profits, there may be some merit in averaging the bid prices obtained from suboptimal bidding and the base-case bids. Such a heuristic can be seen as the action of a participant who wishes to lessen the negative effects of the errors in bid distribution. This action was performed, and once again 168 hours of bidding was performed for scenarios where each of the companies, in turn, submitted the average values to the bid-matching module. Table 5.4 shows the results from these simulations. The data is shown in a format similar to that of Table 5.2, with the columns labeled "1-SUB", "2-SUB", etc., now representing results from bidding the average of the suboptimal and base-case values. The diagonal elements of the new columns are summarized in column 3 of Table 5.5. Comparing this column with column 2 of the same table, it can be seen that the strategic bidding results uniformly in an increase in profits for all the companies, when compared to the base case.

Table 5.4 Modified polynomial model – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	19,649	19,931	19,417	19,011	19,493	19,608	19,273	19,451	18,683
2	19,212	19,227	20,320	19,212	19,221	19,212	19,263	19,227	19,212
3	13,714	13,618	13,689	14,103	13,749	13,726	13,618	13,708	15,247
4	8,118	8,152	8,127	8,118	8,465	8,117	8,117	8,118	8,119
5	83,407	83,123	82,380	80,703	82,426	83,423	81,674	82,964	79,794
6	36,440	36,440	36,521	36,440	36,441	36,441	38,690	36,454	36,440
7	11,784	11,784	11,785	11,784	11,784	11,784	11,792	12,433	11,784
8	17,644	17,547	17,593	19,190	17,631	17,643	17,537	17,611	18,667
Tot.	209,969	209,825	209,835	208,565	209,212	209,958	209,967	209,969	207,949

Whether the above heuristic is the best one for each company can only be determined by performing extensive simulations of different scenarios for each company. But the analysis presented here is to show that heuristic tuning of suboptimal bids is possible for polynomial modeling. The heuristic suggested here is that of averaging the suboptimal



bid with the marginal cost. This action can be interpreted as the actions of a company that is aware of the limitations of the information used by the strategy, and attempts to compensate by making the bid “more conservative” by averaging it with the most conservative bid (from the point of view of acceptance probabilities) – the marginal cost. This action could be further refined by using a weighted average of the two bids, instead of a simple average. The actual weights could be tested by trial and error, and simulation. Again, such a task is very time consuming and has not been pursued in this research. Future research in this direction could prove interesting.

Polynomial modeling thus seems to be a promising way to include market information into the bidding process. It is a relatively fast, and conceptually simple way to model competitors, and the preliminary simulations shown in this section indicate that when heuristically tuned, the bids determined in this way can improve upon the marginal cost bidding benchmark.

Table 5.5 Modified Polynomial model – suboptimal bidding profits in \$.

<i>Company</i>	<i>Scenario</i>	
	INIT	MODSUB
1	19,649	19,931
2	19,212	20,320
3	13,714	14,103
4	8,118	8,465
5	83,407	83,423
6	36,440	38,690
7	11,784	12,433
8	17,644	18,667

Even though it has its advantages, some disadvantages were discovered in polynomial modeling. These are summarized as follows:

- A relatively high degree (20) polynomial was required before a reasonable fit to the CRF curve could be obtained. This results in multiple values for suboptimal

bids, which must be evaluated.

- Because of the fact that a polynomial fit does not always satisfy the property of the CDF that requires it to be between 0 and 1, erratic solutions sometimes result. This is especially true when probability of acceptance is close to zero (sell bid price is high or buy bid price is low). Thus, sanity checks may have to be implemented to ensure that the bid prices resulting from the model make physical sense.
- Since the polynomial model does not have a standard shape associated with it, heuristics are hard to develop that could take advantage of other market information, such as high and low bids, most likely bid, etc., should such information become available to the participant.

The above disadvantages do not exist, at least theoretically, for incomplete-beta modeling. The next section outlines the implementation and testing of that model.

## 5.5 Incomplete-Beta Model

Using the CRF curves generated from the base-case transaction prices, incomplete-beta function curve fits were obtained. As in the previous section, the prices were again divided into on-peak and off-peak prices. The curve fit was achieved by using SPSS version 6.1.1, on a DEC Alpha workstation. The nonlinear regression required for the curve fit was achieved by using the sequential programming option in the package. This option was found to have better convergence properties than the Levenberg-Marquardt algorithm option. Based on this curve fit, two sets of coefficients were obtained, one each for on-peak and off-peak price distributions. These values were substituted into Equations 3.27 and 3.28 which are the objective functions of the buyer and the seller respectively. The value for  $m$ , the maximum likely bid were assumed to be the value of the maximum of the transaction prices occurring in the on-peak or off-peak periods. The

resulting expression was numerically maximized<sup>6</sup> by searching between 0 and 1 for the normalized suboptimal bid, that maximizes the objective function. The numerical maximization, and the required integration of the incomplete-beta function were performed by invoking C functions available in [65]. The maximization algorithm was a modified minimization algorithm known as Brent's method. Generating suboptimal bids for the largest company, company 5, took only 15 CPU seconds on the HP C110 workstations. Thus, the suboptimal bid development implementation using incomplete-beta model is very efficient, compared to the polynomial model.

The resulting bids were assumed to be submitted by each company, in turn, to the bid-matching module. The profits from the simulations of various scenarios, similar to those described in the previous section, are shown in Table 5.6.

Table 5.6 Incomplete-beta weekly model – all-participant profits in \$.

Co.	<i>Scenario</i>								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	19,649	1,306	18,060	18,303	18,859	43,906	16,595	18,073	18,439
2	19,212	19,017	27,393	19,166	19,252	16,392	19,647	19,371	19,141
3	13,714	12,482	13,330	4,503	14,054	6,748	12,789	13,397	17,801
4	8,118	8,056	8,373	8,042	9,786	5,480	8,652	8,240	8,020
5	83,407	94,524	73,954	77,630	79,491	41,130	65,053	77,737	75,413
6	36,440	36,087	36,854	36,329	36,440	31,453	53,597	36,784	36,313
7	11,784	11,648	11,816	11,777	11,784	10,778	11,838	17,620	11,752
8	17,644	16,020	17,100	21,784	17,975	7,609	16,642	17,263	7,085
Tot.	209,969	199,143	206,884	197,537	207,644	163,500	204,816	208,488	193,966

The summary of the data in this table is shown in Table 5.7, in column 3, labeled "WEEKLY". Column 1 of this table shows the profits from the base case simulation described in the previous section. By the comparing the corresponding elements of the two columns in each row, we can see the effect of the strategy on the bidding profits of the participants. Unlike the polynomial modeling case, incomplete-beta modeling by assuming weekly classification of price periods results in negative effects on the profits

<sup>6</sup>Before maximization was performed, the concavity conditions presented in Section A.2.2 were checked

of companies 1, 3, 5 and 8. The profits of the other companies shows an increase over the base case.

Further, it can be seen from columns 3 and 7 of Table 5.6, that when one of the predominantly selling companies 1 and 5 employ the suboptimal strategy, the other company has an increase in profits. Upon examination of the matches (not shown), it was found that this was because the incomplete-beta model parameters under-estimated the bids of the competing company, and thus resulted in a large number of bids being rejected by the bid-matching module for the company employing the strategy. This resulted in the other company virtually cornering the market for energy, leading to the observed profit distributions. These results suggested that perhaps the suboptimal bidding model was not detailed enough in terms of modeling different distributions of competing bids for different periods.

Table 5.7 Incomplete-beta weekly  
model – suboptimal  
bidding profits in \$.

<i>Company</i>	<i>Scenario</i>	
	INIT	WEEKLY
1	19,649	1,306
2	19,212	27,393
3	13,714	4,503
4	8,118	9,786
5	83,407	41,130
6	36,440	53,597
7	11,784	17,620
8	17,644	7,085

Based on the results observed, the next step was to incorporate a more-detailed bid distribution model. The base-case transaction prices were now divided into 7 different days of the week, which were further sub-divided into on-peak and off-peak prices, and CRFs were constructed. These CRFs were then curve-fitted with incomplete-beta functions, using SPSS. Thus, we now modeled 14 different sets of incomplete-beta function

parameters in the 168 hour simulations. The profits from the simulations are shown in Table 5.8, and are summarized in column 3 of Table 5.9, labeled "DAILY". It can be observed from the latter table, that the negative effects on profits persist for companies 1, 3, 5, and 8, although the magnitude of this effect is less compared to the weekly model (column 3, labeled "WEEKLY"). Also, the increase in profits for the other companies is more than in the case of weekly modeling.

Table 5.8 Incomplete-beta daily model – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	19,649	3,833	18,160	18,153	18,983	43,803	16,804	18,122	18,234
2	19,212	19,017	29,148	19,147	19,252	16,551	19,627	19,365	19,122
3	13,714	12,594	13,388	8,068	13,977	9,000	12,996	13,445	17,602
4	8,118	8,059	8,352	7,997	10,382	5,625	8,577	8,232	7,987
5	83,407	94,430	74,099	78,085	79,757	42,429	65,185	77,747	76,097
6	36,440	36,085	36,854	36,291	36,440	31,683	56,474	36,784	36,282
7	11,784	11,647	11,816	11,777	11,784	10,848	11,838	18,375	11,741
8	17,644	16,121	17,090	21,833	17,854	9,576	16,728	17,321	12,121
Tot.	209,969	201,790	208,909	201,356	208,433	169,519	208,233	209,394	199,189

Table 5.9 Incomplete-beta daily models – suboptimal bidding profits in \$.

Co.	Scenario			
	INIT	DAILY	AVERAGE	SPREAD
1	19,649	3,833	9,899	20,310
2	19,212	29,148	24,782	24,472
3	13,714	8,068	13,483	12,915
4	8,118	10,382	10,274	9,816
5	83,407	42,429	80,097	84,418
6	36,440	56,474	47,778	47,310
7	11,784	18,375	15,371	15,428
8	17,644	12,121	18,180	17,421

This suggests that a more detailed model for competing bid distributions could improve the effectiveness of the suboptimal strategy. It would be possible to explore this procedure further by incorporating an even more detailed model, for example, several more than just two price periods per day. However, collating prices, and obtaining

incomplete-beta function fits to CRFs is currently implemented manually, and is a somewhat laborious process even for the less detailed models presented so far. So, we did not attempt to study very detailed price period models.

In spite of the apparent improvement in the effectiveness of the strategy, we can still implement heuristics to further fine tune the suboptimal bids. The first kind of heuristic attempted was identical to the one presented for the polynomial model, i.e., averaging the suboptimal bids with the base-case bids. Results from using these average bids in simulations are shown in Table 5.10.

Table 5.10 Incomplete-beta average model – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	19,649	9,899	18,820	18,670	19,331	33,742	17,961	18,764	18,465
2	19,212	19,042	24,782	19,169	19,244	17,544	19,450	19,315	19,158
3	13,714	12,523	13,536	13,483	13,666	9,138	13,328	13,621	16,539
4	8,118	8,045	8,167	8,050	10,274	6,660	8,145	8,116	8,040
5	83,407	92,906	78,651	79,968	81,537	80,097	74,267	80,552	78,726
6	36,440	36,126	36,759	36,347	36,440	33,405	47,778	36,681	36,349
7	11,784	11,657	11,815	11,777	11,783	11,183	11,838	15,371	11,760
8	17,644	16,103	17,347	20,492	17,535	10,004	17,172	17,510	18,180
Tot.	209,969	206,304	209,880	207,959	209,813	201,777	209,944	209,934	207,221

Here, each company is assumed to (in turn) bid the average of the bids obtained by the suboptimal bidding strategy using daily incomplete-beta function models, and the base case, marginal cost bids. The data shown is in the usual format, and is summarized in column 4 of Table 5.9.

By comparing this column with columns 2 and 3 of the earlier Table 5.7, and column 3 of Table 5.9, it can be observed that while the negative effects still persist for company 1, 3, and 5, the magnitude of this effect is markedly reduced. Company 8 now shows an *increase* in profits. The other companies continue to show an increase in profits. Thus, the heuristic does decrease the negative effects of the strategy. This averaging can once again be interpreted as the action of a participant who wishes to submit a more conservative bid. In fact, this interpretation is supported by the *decrease* in the

magnitude of the increase of profits of companies 2, 4, 6 and 7, when compared to their performances without employing the heuristic. In other words, these companies now submitted needlessly conservative bids, and lost some the benefits from the suboptimal strategy. Achieving the right balance of conservatism and strategy thus is a key to designing the best strategy for a company. Such a balance could be possibly discovered by performing extensive simulations for a variety of scenarios.

The above heuristic, while attractive in its simplicity, fails to incorporate one other advantage of incomplete-beta modeling. This advantage is the fact that the relatively small number of parameters of the model (3), suggest a direct way of compensating for the errors introduced by using transaction prices as a proxy for the buy and the sell bids of competitors. To investigate this advantage further, we need to investigate the sensitivity of the suboptimal bid to changes in the parameter values. Unfortunately, no closed form solution exists for the maximum of the objective function given by Equations 3.27 and 3.28. Therefore, mathematical sensitivities to parameters are hard to derive. However, numerical calculations can be performed to examine the behavior of the suboptimal solution as a function of the three parameters of the incomplete-beta model. These parameters are the shape parameters,  $a$  and  $b$ , of the incomplete-beta function, and the estimated maximum likely bid,  $m$  which is used to normalize the search space for the suboptimal bid. Figures 5.2, 5.3, and 5.4 show the variation of the suboptimal bid with  $m$ ,  $a$ , and  $b$  respectively.

From these figures, it can be seen that the suboptimal bids for both a buyer and a seller increase with an increase in  $m$  and  $a$ , while the bids decrease with an increase in  $b$ . Now, the participant can use these properties to directly adjust the suboptimal bids, instead of taking averages with the base-case bids. Using the sensitivity of the bid to the parameters  $a$  and  $b$  will involve changing the curve fit obtained by regression, and is a more complex operation to interpret physically. This was not attempted in this research.

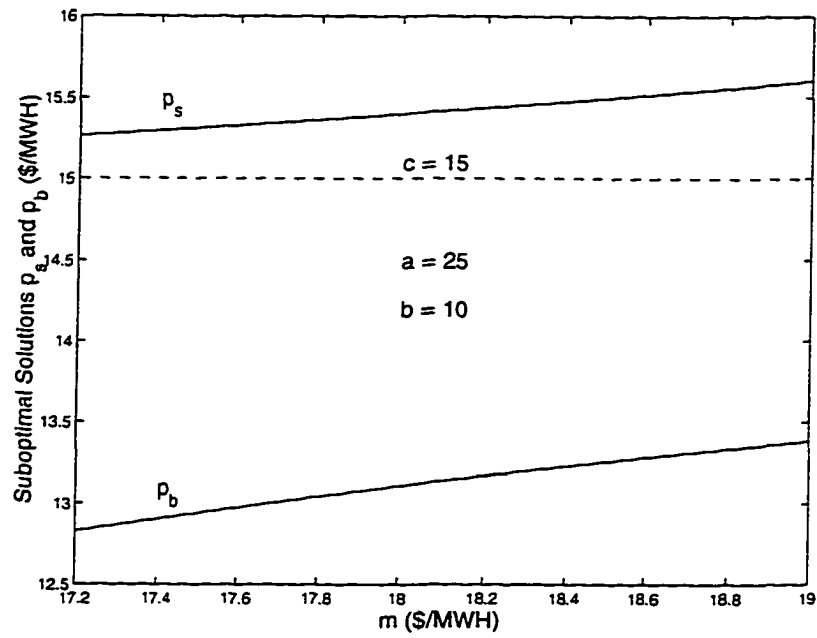


Figure 5.2 Sensitivity of suboptimal solution to  $m$ .

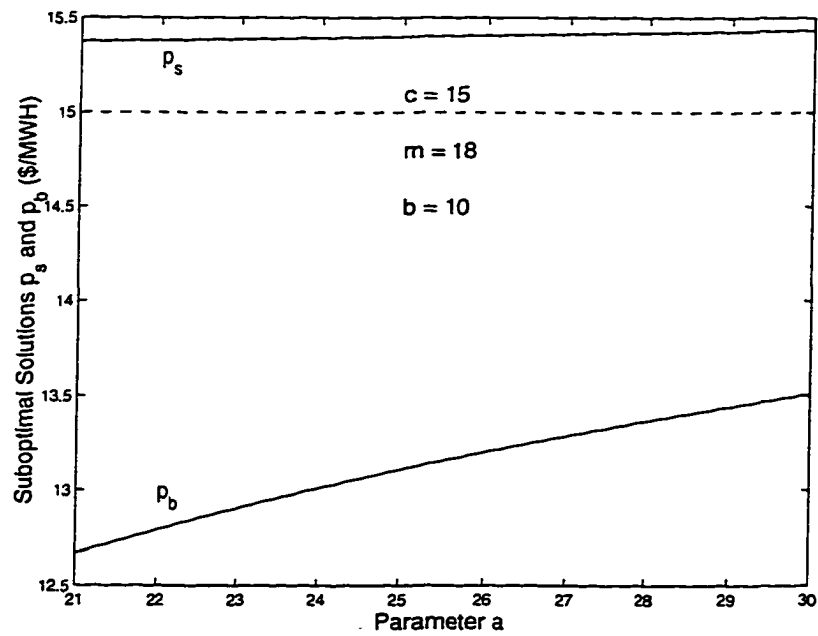


Figure 5.3 Sensitivity of suboptimal solution to  $a$ .



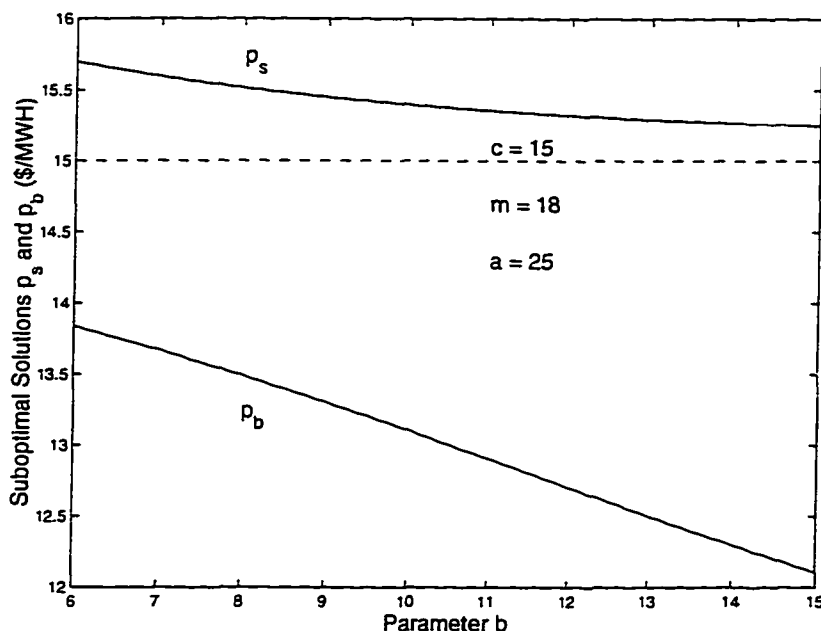


Figure 5.4 Sensitivity of suboptimal solution to  $b$ .

The parameter  $m$ , however, is a normalizing factor, and can be interpreted as the participant's estimate of the largest bid that could occur in the competing bid distribution. Now, we assume that the participant uses transaction prices to obtain  $a$  and  $b$  through regression. Then, instead of using the largest occurring transaction price to directly represent  $m$ , as in the previous cases, the participant changes  $m$  as follows: the values for  $m$  for the buy bid distributions are all increased by 4 \$/MWH, and the values for  $m$  for the sell bid distributions are decreased by 5 \$/MWH. In other words, the participants would like to impose the heuristic tuning that there is a 9 \$/MWH spread between the highest possible buy bid and the highest possible sell bid (since  $m$  is the normalizing factor for the sell bid case also, we still interpret it as the *highest* possible sell bid, and not the lowest possible sell bid). The particular numbers were chosen by (unfairly) looking at bid data and selecting the adjustment to be close to the actual bid distributions. However, it is reasonable to expect that a participant with some experience with the market, or some form of market intelligence, would have an

opinion of the magnitude of this spread, and could include this opinion into the heuristic tuning by adjusting  $m$  as above. The effect of this adjustment would be to increase the suboptimal buy bids and to decrease the suboptimal sell bids. In other words, the participant's strategy now is modified to recognize a "stronger" competition than reflected by transaction prices alone.

Such modifications in the values of the parameter  $m$  were made, and the suboptimal bids were generated for each company. The resulting sell/buy bids displayed the expected decrease/increase. These bids were submitted to the bid-matching module, in turn, and the results from the bidding are shown in Table 5.11. These results are summarized in column 6 of the earlier Table 5.7. Upon comparison of the values in this column with the corresponding values in column 1, it can be seen that the strategy results in an increase in profits over the base case for all the companies except company 3 and company 8. This shows that it is possible to directly modify the bids generated by the suboptimal strategy by adjustments the parameters of the bid distribution heuristically. The fact that the strategy still resulted in a decrease of profits for company 3 and company 8 can be explained by the reasoning that the adjustments were not sufficient in those cases to mitigate the effect of rejected bids.

Table 5.11 Incomplete-beta spread model – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	19,649	20,310	18,845	18,836	19,335	19,665	18,033	18,701	18,505
2	19,212	19,252	24,472	19,212	19,249	19,118	19,471	19,325	19,212
3	13,714	13,309	13,505	12,915	13,680	13,483	13,272	13,603	15,860
4	8,118	8,220	8,161	8,117	9,816	8,076	8,173	8,115	8,119
5	83,407	82,899	78,952	79,694	81,624	84,418	74,673	80,518	78,255
6	36,440	36,435	36,775	36,440	36,441	36,225	47,310	36,713	36,440
7	11,784	11,783	11,816	11,784	11,784	11,756	11,838	15,428	11,784
8	17,644	17,334	17,295	19,828	17,567	17,202	17,110	17,497	17,421
Tot.	209,969	209,543	209,824	206,829	209,498	209,947	209,884	209,902	205,598

## 5.6 Variations on Competitor Strategies

In the previous sections, each competing company was assumed to bid its marginal cost, while the company selected to be Company I in the simulation was assumed to bid strategically. This assumption was made to see clearly the effectiveness of the strategy on a particular participant's performance, given that the bidding of the competitors remains essentially unchanged. This approach can be justified as a "first approximation" solution, in the absence of any knowledge of competitor behaviors. Thus, such a strategy can be tested by simulation even in the absence of any other information except a set of recent transaction prices that are expected to repeat<sup>7</sup>. However, if approximate production cost data is available for competitors, it is conceivable that some companies would model their competitors' bids in a more complex manner than assumed in the previous sections. For example, company 1 might wish to model the key competitor for selling, company 5, as a player who also employed the suboptimal bidding strategy that company 1 itself uses. Alternatively, buying companies may model selling company bids by incorporating a suboptimal strategy, while testing their own bidding strategies for buying.

An exhaustive analysis of all possible combinations is too time consuming to be performed within the scope of this research. However, a set of "extreme case" simulations were performed. In these simulations, each company was assumed to use the data from the base-case simulation to generate suboptimal bids by using each of the strategies illustrated above by simulations. These bids were *simultaneously* used to test the relative effectiveness of these strategies. the results are shown in Table 5.12.

The arrangement of results in this table is similar to the earlier tables showing all the participant' profits. The second column, labeled "INIT" shows the profits from the base

---

<sup>7</sup>If such a set is unavailable, participants could use a *forecast* of transaction prices. Although this seems like uncertain information on which to base bidding, traditionally, utilities have based unit commitment models on forecasted loads. Therefore, the problem of uncertainty in supply and demand conditions is not new.

case simulation. The subsequent columns labeled "POL" and "POL-AV", stand for the cases where *all* participants submit, respectively, suboptimal bids from the polynomial model, and the polynomial model with averaging heuristics employed. Columns 5, 6, 7 and 8 show the results when *all* participants submit suboptimal bids generated by employing, respectively, the incomplete-beta strategy with the weekly model, the daily model, the daily model with averaging heuristics, and the daily model with the assumed spread in the parameter  $m$ . These are labeled "BET-WKL", "BET-DLY", "BET-AV", and "BET-SPR", respectively.

Table 5.12 Simultaneous strategy simulations – all-participant profits in \$.

Co.	Scenario						
	INIT	POL	POL-AV	BET-WKL	BET-DLY	BET-AV	BET-SPR
1	19,649	16,059	17,480	4,016	7,478	14,980	14,704
2	19,212	21,720	20,421	21,180	24,888	22,833	24,944
3	13,714	16,284	15,977	34	697	7,547	14,982
4	8,118	9,337	8,526	1,246	4,583	8,437	10,264
5	83,407	62,965	71,966	30,062	34,110	66,409	55,391
6	36,440	41,060	38,750	42,333	49,413	43,840	47,577
7	11,784	13,140	12,462	16,553	17,420	14,634	15,857
8	17,644	20,730	20,508	282	635	8,578	19,076
Tot.	209,969	201,300	206,092	115,711	139,227	187,262	202,798

It can be seen from this table that the effects of the strategies on individual, as well as total profits in the system is far more complex than in the case when only one participant is assumed to be using the strategy.

In the case of the polynomial strategies, using the polynomial strategy increases the profits of all the predominantly buying participants, companies 2,3,4,6,7 and 8, when compared to the base case profits of column 1. Companies 2 and 5, the primary sellers in the system, lose profits, when compared to the base case. For the polynomial strategy with heuristic tuning, the general pattern of improving individual profits upon heuristic tuning is not observed. Compared to the base case, profits for the buyers, companies 2, 3, 4, 6, 7 and 8 are still higher. Profits for the sellers, companies 1 and 5 are lower than the base case, but are not as low as without heuristic tuning. Also, total system

profits for the strategic bidding cases are not as high as in the base case. This is understandable, since the base case is the case where all players bid marginal costs, and such a solution is the optimal solution from the point of view of total system savings. However, as we improve the bidding strategies heuristically, we find that the total system profits do increase. This can be interpreted as the market participants learning to guess competitor's behavior better, and thus the total system profits move towards the base case of perfect competition.

Comparing the incomplete-beta models with the base case, we find a similar situation, with the buyers, in general, improving their performances with strategic bidding, at the expense of the sellers. Again, as we improve the detail of the modeling, from weekly model, to daily model, to daily model with averaging and finally to daily model with a spread assumed, we find that total system profits improve towards the base case.

While it is interesting to observe that the simultaneous strategies trend towards the perfectly competitive base case, the goal of the strategies, and the function of the simulator is not to achieve competitive equilibrium. Instead, it is to test the effectiveness of each strategy in enhancing the performance of an individual participant, under various conditions. This can be seen by comparing the columns labeled "POL-AV" and "BET-SPR", the most "sophisticated" strategies studied in this section, with the base-case column, labeled "INIT". Clearly, although the *total* system profits approach the competitive levels in all three cases, the *distribution* of profits among the market participants is different in each case. Thus, the effectiveness of the strategy has been different for each player. Also, in performing these simulations, it has been assumed that each participant uses the same bid distributions, with the same heuristic tuning, to calculate the suboptimal bids. This might not occur in real life situations. Therefore, the simulations and analyses presented in this section are not an exhaustive or even realistic representation of real-life scenarios. Rather, they are simplified versions, that could be improved upon by participants, if they so desire, by including detailed models for competitors, and by

performing a number of simulations.

## 5.7 Effects of the Transmission Network Module

In this section, the results of simulations are presented, which included transmission network modeling to a limited extent, as outlined in Section 4.2.2. The companies modeled in this section so far were now assumed to be sited at the various generating buses, as shown in the earlier Figure 5.1. Four sets of simulations were performed, under two different line loading assumptions.

### 5.7.1 Light Line Loading Conditions

Under the first assumption, the maximum allowable flow on each line in the system was assumed to be 200 MW. This assumption implies a relatively lightly loaded transmission system. In other words, if a flow of more than 200 MW occurred in any of the lines as a result of a match, this match would be rejected. For this assumption, two sets of simulations were performed. One simulation was a repeat of the base-case simulation of the previous sections, with participants bidding marginal costs. The other set of simulations involved the participants (in turn) submitting the suboptimal bids developed by assuming a modified parameter  $m$  for the incomplete-beta modeling, which was discussed in the last part of the previous section. The results are shown in Table 5.13, and are arranged in the usual fashion.

It can be seen from column 1 of this table that the base-case profits of all the companies were less than the base-case profits when transmission was not considered. The results are summarized in Table 5.14. In this table, profits from bidding the base case and from bidding the strategic bids, are shown in columns 2 and 3, labeled “INIT-LGT” and “SPR-LGT” respectively. A comparison of the two columns shows the effect of employing the suboptimal bidding strategy with heuristic tuning of the distribution

Table 5.13 Light line loading – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	7,798	8,083	68,32	7,800	7,840	7,908	6,081	6,922	7,808
2	12,462	12,460	18,432	12,462	12,469	12,395	11,309	11,944	12,462
3	1,137	829	917	1,104	1,101	1,137	977	1,093	1,140
4	167	180	181	167	44	160	36	166	167
5	21,446	20,879	16,830	21,460	21,445	21,656	14,703	18,427	21,482
6	10,876	10,820	10,279	10,848	10,876	10,629	15,219	11,594	10,875
7	5,847	5,883	5,731	5,875	5,847	5,827	5,995	9,226	5,847
8	2,356	2,213	2,261	2,350	2,270	2,356	1,077	2,139	2,356
Tot.	62,091	61,352	61,466	62,069	61,895	62,071	55,400	61,514	62,141

Table 5.14 Transmission modeling – suboptimal bidding profits in \$.

Co.	Scenario			
	INIT-LGT	SPR-LGT	INIT-HVY	SPR-HVY
1	7,798	8,083	7,185	7,315
2	12,462	18,432	12,219	17,595
3	1,137	1,104	1,102	1,067
4	167	44	142	38
5	21,446	21,656	20,570	20,780
6	10,876	15,219	10,686	15,001
7	5,847	9,226	5,847	8,797
8	2,356	2,356	2,223	2,223

parameters. It can be seen that companies 3 and 4 experienced a decrease in bidding profits, company 8 has the same profit as the base case, while all the other companies experienced an increase in profits compared to the base case. The profits shown in the tables take into account the reduction in profits because of the transmission costs. Also, upon examination of the output files from the bid-matching modules, it was observed that the volume (the number of accepted transactions) was lower than the case where transmission was not considered, with 18 proposed transactions being rejected because of line flow limitations, and 3770 proposed transactions being rejected because transmission costs were not justified by the energy cost savings. Another interesting aspect of modeling transmission is the fact that, since we model transmission cost by the incremental MW-mile cost impact, there are transactions for which the transmission costs are *negative*. In the network module, the transmission costs for an accepted match were assumed to be evenly split between the buyer and the seller. In the lightly loaded case, the effect of the strategies is very similar to the effects seen without transmission modeling. The fact that company 4 experienced a decrease in profits can be explained from Figure 5.1. In this figure, company 4 is located at bus 29, relatively far from companies 1 and 5, the primary sellers, who are located at buses 4 and 6 respectively. This distance results in the rejection of a large number of matches between company 4 and the sellers, on the basis of insufficient savings to justify transmission costs, even in the base case. In the suboptimal case, the energy cost savings would be even less because the strategy decreases the buy bids, and thus more transactions would be rejected between company 4 and the sellers. For company 8, the only transactions that were allowed by the network module for the suboptimal strategy case, are the ones that by default, are equal to the marginal cost bids, and so the profits for this company are identical to the base case. Once again suboptimal bidding with incomplete-beta modeling, with adjusted  $m$ 's was simulated, with the new line limits in place. Results are given in Table 5.13 and summarized in columns 2 and 3 of Table 5.14.



### 5.7.2 Heavy Line Loading Conditions

Under the second assumption, each line was now assumed to have a maximum flow limit of only 100 MW. Simulations from the previous case were repeated with these new limits in place. The base-case profits are given in column 2 of Table 5.15. These are less than the base-case profits for the lightly loaded case. For this case, 849 bids were rejected because of line flow limitations, and 3026 were rejected because transmission costs are not justified by energy cost savings. Base-case and suboptimal bidding simulations were again performed for each company and the results are shown in Table 5.15, and summarized in the earlier Table 5.14, labeled "INIT-HVY" and "SPR-HVY" respectively. Comparing these columns (4 and 5) of this latter table we find that the effects on the profits are similar to the lightly loaded case.

The above two sets of assumptions are by no means an exhaustive set of transmission conditions under which bidding strategies can be tested. However, they have been included to illustrate some of the effects that transmission loading has on bidding profits.

Table 5.15 Heavy line loading – all-participant profits in \$.

Co.	Scenario								
	INIT	1-SUB	2-SUB	3-SUB	4-SUB	5-SUB	6-SUB	7-SUB	8-SUB
1	7,185	7,315	6,202	7,183	7,182	7,295	5,785	6,319	7,195
2	12,219	12,206	17,595	12,190	12,225	12,152	11,309	11,800	12,219
3	1,102	821	914	1,067	1,056	1,103	978	1,034	1,106
4	142	174	181	142	38	136	36	141	142
5	20,570	20,391	15,981	20,558	20,572	20,780	14,305	17,629	20,607
6	10,686	10,634	10,255	10,661	10,686	10,439	15,001	11,524	10,686
7	5,847	5,883	5,731	5,875	5,847	5,827	5,995	8,797	5,847
8	2,223	2,080	2,081	2,217	2,130	2,223	1,015	1,984	2,223
Tot.	59978	59508	58943	59897	59740	59958	54429	59232	60028

## 6 SCHEDULING-BASED STRATEGIES

In Chapter 3, the strategies described were primarily concerned with determining the optimal price to bid, *given a cost or value of generation*, such that other participants' bidding behavior was incorporated. The cost was assumed to be a simple number already available from, say, economic dispatch calculations. However, in this chapter, scheduling considerations are analyzed, that lead to the calculation of this cost. In the first part of this chapter, a qualitative treatment of scheduling factors that may affect bidding is presented. This treatment is to provide a broad scope for the sample numerical examples that follow in the third part of this chapter. Complete analysis of all the factors is beyond the scope of this research work. However, analysis for some typical scenarios is presented.

The second part of this chapter provides background on utility functions as a means of incorporating risk preferences into the participants' bidding strategies. Again, the goal of the presentation is to provide a scope for using utility theory results in generation bidding strategies, as opposed to providing an exhaustive treatment of utility theory itself. References are provided on this subject for the interested reader.

### 6.1 Scheduling Considerations in Bidding

In Section 3.3.1, and in the subsequent analyses presented in Chapter 3, the implicit assumption made was that the commitment schedules of the generating units were pre-determined from native loads or pre-existing firm contracts. In this section, we provide some reasons that lead to generating unit commitment schedules being changed from

the “base case”. Table 6.1 lists the different types of schedule changes that could occur, in the columns. A ‘√’ in a cell indicates that this schedule change could occur as a result of the reason listed in the corresponding row. An ‘X’ indicates that it is uncommon for this schedule change to occur as a result of this reason.

Table 6.1 Reasons for changing commitment schedule.

<b>REASON</b>	<b>Startup</b>	<b>Delayed Shutdown</b>	<b>Shutdown</b>	<b>Delayed Startup</b>
Generation requirements	√	√	√	√
Reliability requirements	√	√	√	√
Maintenance requirements	√	√	√	√
Environmental requirements	X	X	√	√
Fuel considerations	√	√	√	√
Efficiency considerations	√	√	√	√
Market conditions	√	√	√	√
Secondary effects	√	√	√	√

The following are brief explanations of the reasons:

- *Generation requirements:* The forecasted load used in the initial unit commitment might be too high or too low, causing a unit status change to be required or considered. Also, changes in the system schedule because of other reasons (given below) could result in a change in the generation requirement. This could be as a result of changing weather conditions or power system events including outages, load forecast errors or a unit returning from maintenance earlier than planned.
- *Reliability requirements:* Some of the units might be required to be started or kept running for reasons such as spinning reserve or to provide reactive support. Scheduled transactions might have to be reduced or terminated, or generating units might have to be shutdown because of transmission limitations imposed by reliability requirements. Such a need could arise at short notice, also because of changes in weather conditions or equipment outages.

- *Maintenance requirements:* Unforeseen changes in maintenance schedules and testing procedures might require a change in unit status. The changes in maintenance schedules might be a delay or an advancement in a generating unit's availability, or an increase or decrease in the expected capacity at which the unit is available. Thus, it could result in any of the four scheduling decisions being considered.
- *Environmental requirements:* Load fluctuations might lead to unforeseen level of emission amounts, causing the shutdown or delayed startup of thermal units. River water cooled units might require shutdowns because of heat exchange limits. Loss of a scrubber or other air emission control device might also affect the operation of thermal units. Other examples include emergency situations in nuclear units and water availability or flow requirements in hydro units.
- *Fuel considerations:* Take-or-pay requirements might force thermal units to startup or delay shutdown. Fuel network events, such as transportation disruptions, might force thermal units to shutdown or delay their startup. Fuel spot-market conditions and related decisions could also affect the operation of fossil units.
- *Efficiency considerations:* Potential savings from changing commitment status of units, other than those identified by the scheduling algorithm might arise because of changed system or market conditions.
- *Market Conditions:* Supply, demand and price conditions in the spot electricity market might provide incentive to change the commitment status.
- *Secondary Effects:* A change in status of one unit because of one or more of the above reasons might result in a secondary effect on system conditions, that might require a further change in status of other units. For example, a unit startup for reliability requirements might result in an excess generating capability or emission limit violation, either of which could result in the shutdown of some other unit.

## 6.2 Factors To Be Considered in Scheduling Decisions

In addition to the above factors, which arise at the system level, the following generating unit level characteristics should be considered relevant to bidding decisions:

- Unit type and size
- Unit condition
- Current status
- Minimum and maximum power output
- Incremental heat rate curves
- Minimum up and down times
- Startup cost components and time requirements
- Response times and ramp rate limits
- Reserve contribution
- Fuel types and fuel availability
- Environmental impact
- Geographical location

Also, local transmission conditions may play a significant role in changing generating unit status.

### 6.3 Effects of Schedule Changes on Economics and Operations

In this section, the *effects* of the schedule changes listed in the previous section, on economics and on operations are listed. In Tables 6.2 and 6.3, a '?' means that the schedule change listed in the columns may or may not have the effect listed in the corresponding rows, while a '✓' means that the effect is usually observed.

Table 6.2 Effects of changing commitment schedule on economics.

<b><i>EFFECT</i></b>	<b><i>Startup</i></b>	<b><i>Delayed Shutdown</i></b>	<b><i>Shutdown</i></b>	<b><i>Delayed Startup</i></b>
Changes total production cost	✓	✓	✓	✓
Changes marginal cost	?	?	?	?
Changes selling capability	✓	✓	?	?
Changes buying capability	?	?	✓	✓
Changes environmental requirements and costs	✓	✓	✓	✓
Changes fuel costs	✓	✓	✓	✓
Introduces demand risk	?	?	?	?

Table 6.3 Effects of changing commitment schedule on operations.

<b><i>AFFECTED AREAS</i></b>	<b><i>Startup</i></b>	<b><i>Delayed Shutdown</i></b>	<b><i>Shutdown</i></b>	<b><i>Delayed Startup</i></b>
Maintenance schedules	✓	✓	✓	✓
Reliability margins	✓	✓	✓	✓
Fuel consumption	✓	✓	✓	✓
Emissions	✓	✓	✓	✓
Operational flexibility	✓	✓	✓	✓

### 6.4 Modeling Risk Preference in Scheduling Decisions

Thus far in this dissertation, it has been assumed that the participants under consideration are all expected value maximizers. In other words, the players always seek to optimize expected values of profits, without regard to the magnitude of monetary losses, resulting from the downside to a rejected bid. This does not seem unreasonable when

considering the relatively simple bidding scenarios where the unit commitment is fixed, because there is no downside to a rejected bid <sup>1</sup>. However, when scheduling considerations such as a change in commitment are incorporated, then situations arise where downsides exist in the form of additional system costs incurred or penalties, as will be illustrated in later sections. In such situations, expected value maximization alone might be insufficient to model the participants' behaviors. When there are distinct downsides present in bidding outcomes, utility theory provides procedures to incorporate the risk involved. Before we begin to model this risk, the following section presents some of the factors that may cause risk in bidding outcomes.

#### 6.4.1 Classification of Risk

Several definitions of risk are available. One such definition of risk is, the effect of a certain event on an objective function multiplied by the probability of the event. In this research, however, we define bidding risk as follows:

*Bidding risk is the product of monetary losses from a certain event, and the probability of occurrence of this event. If several mutually exclusive events exist, that may cause a monetary loss from bidding, then the total risk is the sum of the risks from each such event.*

One type of event that could cause monetary losses is the rejection of a bid by the broker if a schedule change has been made by the participant, who expects the bid to be accepted. In this case, the monetary losses could occur as reduced revenues, unrecovered startup costs, dump power penalties, etc. The probability of occurrence of this event is calculated as the complement of the probability of acceptance. Thus, the risk in this case can be computed.

Another type of risk that could cause monetary losses is the outage of a unit in a

---

<sup>1</sup>In reality, the only downside of a rejected bid is the fee to be paid to the broker, which is implicitly assumed to be small. Even if the fee is substantial, it is reasonable to expect that it would be fixed, and not a function of the bid price. Thus, it does not enter into the objective function at hand.

seller's system, after a bid has been accepted. The probability of outage of the unit could be obtained from unit forced outage rates, which are presumably independent of bid acceptance probabilities. So the risk in this case can also be computed.

The former kind of risk is essentially a risk due to market conditions, i.e., it is a *price risk*. The latter kind of risk is a risk due to generating system conditions, i.e., it is a *production risk*. A third type of risk is that associated with the third-party transmission delivery system. This type of risk is very complex to quantify and analyze, and is beyond the scope of this research work. However, the utility theory approach can be modified to include the risk from transmission system effects if participants have a measure of these effects that they can trust.

#### 6.4.2 Utility Functions

Both kinds of risks described above need to be incorporated in the bidding decisions of participants, in order to obtain a realistic and usable bidding strategy. One possible way to do this is to maximize the expected value of profit minus the risk instead of maximizing expected value of profit. This may or may not lead to a solution that does not truly reflect the goals of the participant. For example, a participant may consider losing a dollar to be worse than gaining a dollar. So, we need some way to incorporate the downside of the bidding also into the profit objective function, while attempting to suitably tradeoff between profit and risk. On the other hand, the participant may not assign the same amount of negative value to losing or gaining a dollar, under all circumstances. For example, a participant who has achieved a large percentage of a given periods profit goals might be more willing to risk the loss of a dollar than he was at the beginning of a given period. Utility functions offer a systematic and rational way to do include such considerations, while allowing the participant to choose his attitudes towards risk, termed as *risk preferences*. Indeed, it can be shown that expected value maximization is a special case of a linear utility function, one that gives equal weightage



to profit and risk.

The utility function, in the context of this research, is a measure of the satisfaction that a participant derives from a certain level of profit, denoted as by the variable *wealth*,  $w$ . The amount of satisfaction, while a non-decreasing function of wealth, need not be linearly dependent on wealth. The advantage of using the utility function is to model the behavior of a variety of rational participants, with different attitudes toward wealth in the presence of uncertainty and risk. In order to incorporate uncertainty and risk into the decision making problem, the participants will be assumed to maximize *expected utility* of profits as opposed to expected value of profits.

Let us consider the following problem involving uncertainty, which is slightly modified from an illustrative example from [66]. A participant has two possible outcomes from a bidding situation, that would result in profits, or wealths, of  $A$  and  $B$  respectively. The probability of the outcome leading to  $A$  is  $p$ , and the probability of the other outcome is  $1 - p$ . Such a situation is called a *lottery* and is denoted by:

$$L = (P, A, B) \quad (6.1)$$

The probability  $P$  basically introduced uncertainty in the wealth outcome. Under such situations, Von Neumann and Morgenstern [67] showed that it is possible to construct a utility function that can be used to model the participant's choice. If  $U(w)$  represents the utility function of the participant, then the expected utility of the lottery  $L$  is given by:

$$E[U(L)] = PU(A) + (1 - P)U(B) \quad (6.2)$$

This is different from the utility of the expected profit from the lottery, which is:

$$U(E[L]) = U(PA + (1 - P)B) \quad (6.3)$$

The above equation is simply the utility of the expected value of profit from the lottery, and maximizing it is identical to maximizing expected value of profit, because

utility functions are monotonically increasing (additional wealth always increases utility, although the amount of increase may vary.). However, an expected utility maximizing participant maximizes the function given by Equation 6.2. The Von Neumann-Morgenstern utility functions can be used to model risk preferences precisely because of this. A participant who maximizes expected utility has different attitudes toward decisions in the face of uncertainty, depending on the level of wealth they have achieved. These differences may not depend on the *total* wealth they possess, i.e., on the absolute size of the participant, but rather, on the *incremental* amount of wealth they have acquired in a given bidding period. In other words, regardless of the size of the participant, they might have different attitudes toward risk, based upon the recent performance in the market.

Now, a person is a *risk averter* relative to a lottery if the quantity in Equation 6.3 is greater than the quantity in Equation 6.2. In other words, the person prefers a certain outcome to an uncertain one with the same expected value. For this to be true, we need to select  $U(w)$  to be concave. It is also possible to model a *risk seeker*, in other words, a participant with the opposite behavior, by selecting a convex  $U(w)$ . However, in this research, we will assume that all participants are risk averse. Now, even though we assume that all participants are risk averse, the *degree* to which they are risk averse may vary depending on a lot of factors. In other words, all participants need not necessarily have the same amount of willingness (or lack thereof) to take risk under all situations. To quantify this degree of risk aversion, the Arrow-Pratt [68] coefficient of absolute risk aversion,  $r$ , is defined as follows:

$$r = -\frac{U''(w)}{U'(w)} \quad (6.4)$$

This measure is positive if the participant is a risk averter<sup>2</sup>. One other detail regarding utility functions will be examined before we relate utility function theory to

---

<sup>2</sup>For a participant who maximizes expected value of profit, as in Chapter 3, this measure is zero. Such a participant is said to be *risk neutral*.

strategic bidding. This is the derivative of the measure  $r$  with respect to wealth. In other words, how does the participant's degree of risk aversion vary with wealth? We would intuitively expect most participants in common decision making situations to be less risk averse as wealth increases. For example, a large, profitable corporation would be less averse to losing \$100,000 than a smaller or less profitable company. Thus it would seem as though a logical choice for a utility function should be one that has  $r'(w) < 0$ . Consider the modified exponential utility function given below:

$$\begin{aligned}
 U(w) &= 1 - e^{-w^{0.5}} \\
 \Rightarrow r(w) &= 0.25 (w^{-1} + w^{-1.5}) > 0 \\
 \Rightarrow r'(w) &= -0.25w^{-2} - 0.375w^{-2.5} < 0
 \end{aligned} \tag{6.5}$$

Clearly, this could be a choice of utility function to model our participants. Such a participant is said to have *decreasing absolute risk aversion* with increasing wealth (DARA). However, with a lack of much knowledge on how participants' risk taking behaviors will be in the deregulated future, we cannot make such a strong assumption. Thus we need to be able to model participants that have *constant*, or perhaps even *increasing absolute risk aversion* with increasing wealth (CARA and IARA). For these, two possible utility functions are the exponential and the quadratic utility functions respectively:

$$\begin{aligned}
 U(w) &= 1 - e^{-w} \\
 \Rightarrow r(w) &= 1 > 0 \\
 \Rightarrow r'(w) &= 0
 \end{aligned} \tag{6.6}$$

$$\begin{aligned}
 U(w) &= w - \alpha w^2 \\
 \Rightarrow r(w) &= \frac{2\alpha}{1 - 2\alpha w} > 0 \\
 \Rightarrow r'(w) &= \frac{4\alpha^2}{(1 - 2\alpha w)^2} > 0
 \end{aligned} \tag{6.7}$$

Thus, we have illustrated the use of different forms of utility functions to model different participants. However, it would be convenient for implementation, and add to the flexibility of modeling, if *one* form of utility function could be used to model all three kinds of risk attitudes, DARA, CARA and IARA. Such a form is the Expo-Power utility function proposed by Saha in [4]. This function is given by:

$$\begin{aligned} U(w) &= \theta - e^{-\beta w^\alpha} \\ \Rightarrow r(w) &= \frac{1 - \alpha + \alpha\beta w^\alpha}{w} \end{aligned} \quad (6.8)$$

where  $\alpha, \beta, \theta$  are positive parameters. By varying these parameters, we can achieve varying degrees of risk aversion. For example, for  $\alpha = 1$ , we get CARA; for  $\alpha = 0.8$  we get DARA, etc. Figures 6.1 and 6.2 illustrate the effects of  $\alpha$  and  $\beta$  on the Expo-Power utility. In Figure 6.1,  $\theta = 1$ , and  $\beta = 4.1667 \times 10^{-4}$ . In Figure 6.2,  $\theta = 1$ , and  $\alpha = 1$ .

Figure 6.3 illustrates the variation of  $r(w)$  with wealth for different values of  $\alpha$ . Again, for this figure,  $\theta = 1$ , and  $\beta = 4.1667 \times 10^{-4}$ .

For the rest of this chapter, the utility function used to model participant risk attitudes will be assumed to be of the Expo-Power form.

#### 6.4.3 Modeling Risk in Strategic Bidding Using Utility Functions

Now, we will relate the utility function concepts described above to the strategic bidding problem. Consider the objective function from Chapter 3, Equation 6.9, repeated here:

$$\begin{array}{cc} \text{Maximize} & \\ p_b & S(p_b)(c - p_b) \end{array} \quad (6.9)$$

This objective function represents an expected value of profit maximizing participant, with no downside to a rejected bid. Now let us assume that the participant is an expected utility maximizer. Then, the objective is modified as follows:

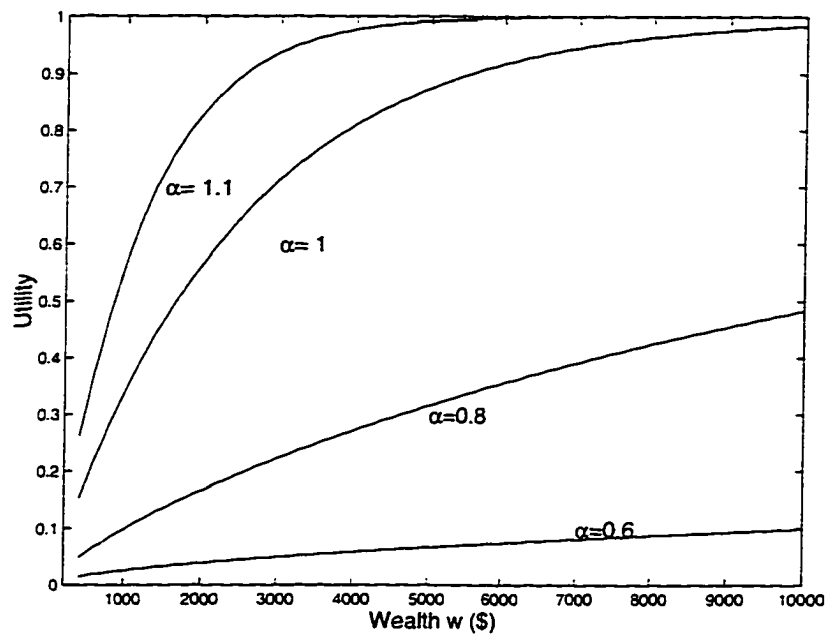


Figure 6.1 Expo-Power utility: variation with  $\alpha$ .

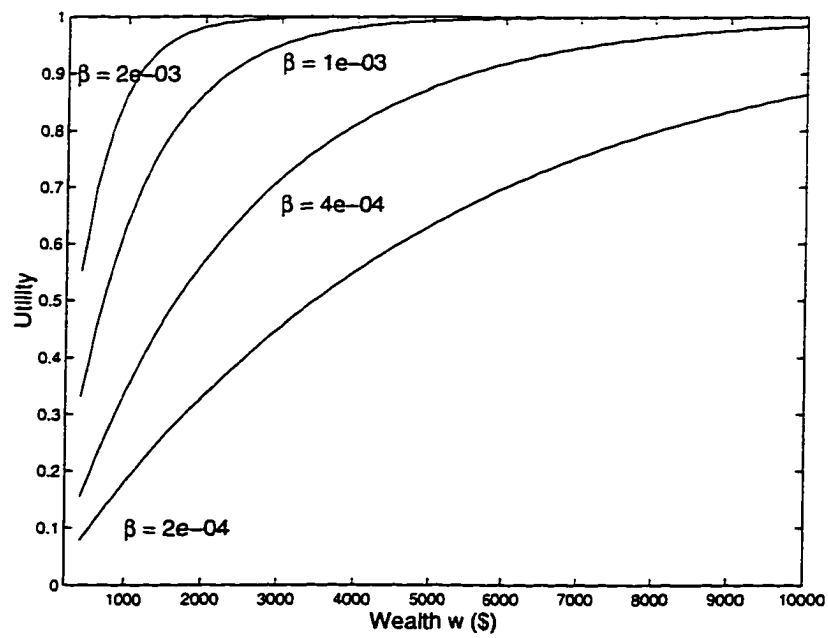


Figure 6.2 Expo-Power utility: variation with  $\beta$ .

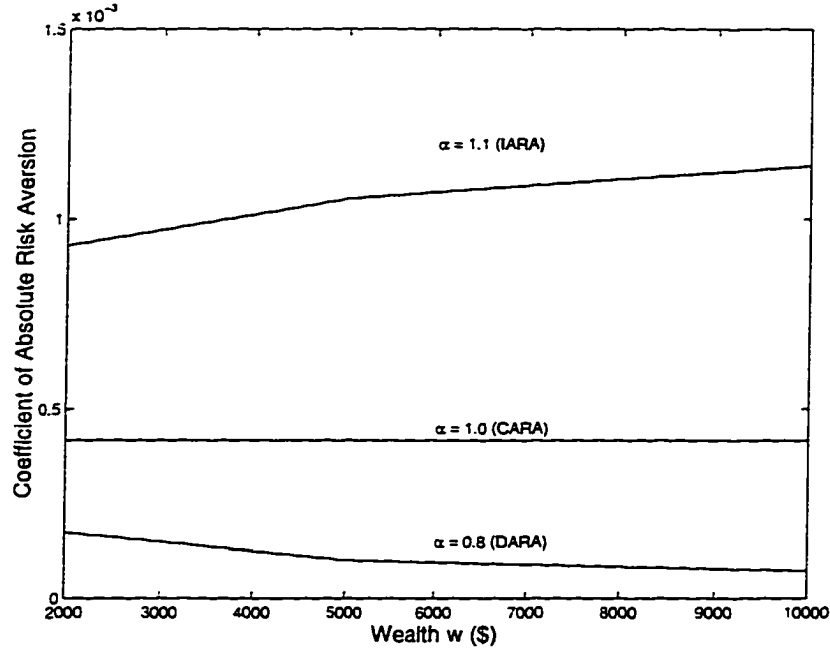


Figure 6.3 Variation of coefficient of absolute risk aversion with  $\alpha$ .

$$\begin{aligned}
 & \underset{p_b}{\text{Maximize}} \quad S(p_b)U(c - p_b) = \\
 & S(p_b) \left[ \theta - e^{-\beta(W + q(c - p_b))^\alpha} \right]
 \end{aligned} \tag{6.10}$$

In this equation,  $\alpha$ ,  $\beta$ , and  $\theta$  are the usual parameters of the utility function.  $c$  and  $p_b$  are the usual cost and bid price values.  $S(p_b)$  is the probability of acceptance of bid price  $p_b$ .  $q$  is the quantity of energy being bid for, i.e., it represents the size of the bid. The new parameter introduced here,  $W$  is an initial wealth parameter, that can be used to represent the current wealth of the participant, at the time the bidding decision is made. It is distinguished from total wealth  $w$  by the fact that it is a constant, and is a parameter selected by the user. Thus overall wealth of the participant, for the purposes of evaluating a bid of size  $q$  is given by:

$$w = W + q(c - p_b) \tag{6.11}$$

$W$  will affect the outcome of the suboptimal bidding procedure, depending on the choice of the other parameters. The reason for including it is twofold:

- $W$  adds to the model flexibility by providing one more way to fine tune risk attitudes.
- A positive  $W$  is required when using non-integral values for  $\alpha$ , in the presence of events that cause a financial loss. This is because, such events result in a negative incremental utility (a negative wealth), and with non-integral  $\alpha$ 's,  $W$  is required to avoid complex number solutions to the suboptimal bidding problem.

Similarly, for a seller, the objective function now becomes:

$$\begin{aligned} \text{Maximize} \\ p_s \quad S(p_s)U(p_s - c) = \\ S(p_s) \left[ \theta - e^{-\beta(W+p_s-c)^\alpha} \right] \end{aligned} \quad (6.12)$$

The suboptimal bidding problem now reduces to maximizing the objective functions given by Equation 6.10 and 6.12. The derivatives of the Expo-Power equation can be easily calculated, and maximization of the concave objective can be performed by any of a number of classical optimization techniques. Examples of such analyses will be presented in later sections of this chapter. In the next section, we examine some scenarios that illustrate what scheduling considerations could affect the bidding process.

## 6.5 Scheduling Considerations – Illustrative Scenarios

### 6.5.1 Change in Generation Requirements

This section illustrates a change in commitment schedule that is mandated by a change in generation requirement. Such a change could arise because of a changed load forecast, weather conditions, etc., which was not foreseen at the initial commitment stage. The following two cases illustrate the options available to the participant when the generating capacity committed for the current hour is too high or too low for the following hour.

1. Generation committed is too low.

The options that might be available to the participant include:

- (a) Additional unit startup (with or without additional bids to buy or sell)
- (b) Additional unit delayed shutdown (with or without additional bids to buy or sell)
- (c) Bid to buy with brokerage
- (d) Attempt to buy or sell through bilateral transactions
- (e) Default on generation requirement with penalty incurred
- (f) Use available interruptible native load contracts
- (g) Combination of above

2. Generation committed is too high.

The options that might be available to the participant are:

- (a) Additional unit startup (with or without additional bids to buy or sell)
- (b) Additional unit delayed shutdown (with or without additional bids to buy or sell)
- (c) Bid to buy with brokerage
- (d) Attempt to buy or sell through bilateral transactions
- (e) Default on generation requirement with penalty incurred
- (f) Use available interruptible native load contracts
- (g) Combination of above



### 6.5.2 An Increase in Reliability Requirements

This section illustrates a change in commitment schedule required for reliability reasons, typically a must-run situation for units, or an increased spinning reserve requirement. The options available to the participant would be:

1. Generation committed in an area is too low
  - (a) Additional unit startup (with or without additional bids to buy or sell)
  - (b) Additional unit delayed shutdown (with or without additional bids to buy or sell)
  - (c) Bid to buy with brokerage
  - (d) Attempt to buy or sell through bilateral transactions
  - (e) Default on generation requirement with penalty incurred
  - (f) Use available interruptible native load contracts
  - (g) Combination of above
2. Generation committed in an area is too high
  - (a) Additional unit shutdown (with or without additional bids to buy or sell)
  - (b) Additional unit delayed startup (with or without additional bids to buy or sell)
  - (c) Bid to sell with brokerage
  - (d) Attempt to buy or sell through bilateral transactions
  - (e) Default on generation requirement with penalty incurred
  - (f) Combination of above

### 6.5.3 A Change in Maintenance Requirements

This section illustrates the cases when generating units that are down for maintenance come on line either later or earlier than expected during the initial commitment stage. It is not unusual for units to have a must-run requirement immediately following maintenance, for testing reasons. Such a status will be assumed in the following cases, so that a change in commitment will be required. This should not be considered as unusual since all units realistically have a minimum run time after startup.

1. Unit maintenance time is longer than expected and unit is unavailable for immediate startup

This could lead to a shortage in online generation from the planned scenario. The options available to the participant would be:

- (a) Other unit startup (with or without additional bids to buy or sell)
- (b) Other unit delayed shutdown (with or without additional bids to buy or sell)
- (c) Bid to buy with brokerage
- (d) Attempt to buy or sell through bilateral transactions
- (e) Default on generation requirement with penalty incurred
- (f) Use available interruptible native load contracts
- (g) Combination of above

2. Unit maintenance time is shorter than expected and unit is started sooner than planned

This could lead to a surplus in online generation if the unit is started. Then, the options available to the participant would be:

- (a) Other unit shutdown (with or without additional bids to buy or sell)
- (b) Other unit delayed startup (with or without additional bids to buy or sell)

- (c) Bid to sell with brokerage
- (d) Attempt to buy or sell through bilateral transactions
- (e) Default on generation requirement with penalty incurred
- (f) Combination of above

#### **6.5.4 Unit Environmental Requirements**

This section illustrates the case where a unit must be shutdown because of environmental reasons, not foreseen at the initial commitment stage. Such a situation might arise because of emission limits being reached, or weather conditions as in the case of river cooled thermal units. Loss of a scrubber or other emission-control device on a thermal unit might also be a cause. Other examples include emergency situations at nuclear units, and water availability or flow requirements for hydro units. The options available to the participant are:

- (a) Specified unit backdown or shutdown (with or without additional bids to buy)
- (b) Specified unit delayed startup (with or without additional bids to buy)
- (c) Other unit delayed shutdown (with or without additional bids to buy or sell)
- (d) Other unit startup (with or without additional bids to buy or sell)
- (e) Bid to buy or sell with brokerage
- (f) Attempt to buy or sell through bilateral transactions
- (g) Default on generation requirement with penalty incurred
- (h) Use available interruptible native load contracts
- (i) Combination of above

### 6.5.5 Unit Fuel Considerations

Fuel considerations in the operation of thermal units are very complex, and often lead to changes in the original schedules of units. Under certain fuel network conditions, fuel suppliers strictly enforce take-or-pay requirements. Thus, participants have to conform to the requirements with short notice, not foreseen in the initial commitment stage. Also, fuel network problems could lead to the shutdown of units. Thus, two kinds of scenarios that could arise are described in the following cases.

#### 1. Increased fuel consumption requirements

The options available to the participant include:

- (a) Specified unit startup (with or without additional bids to sell)
- (b) Specified unit delayed shutdown (with or without additional bids to sell)
- (c) Other unit delayed startup (with or without additional bids to buy or sell)
- (d) Other unit shutdown (with or without additional bids to buy or sell)
- (e) Bid to buy or sell with brokerage
- (f) Attempt to buy or sell through bilateral transactions
- (g) Default on generation requirement with penalty incurred
- (h) Combination of above

#### 2. Decreased fuel availability.

The options available to the participant would be:

- (a) Specified unit shutdown (with or without additional bids to buy)
- (b) Specified unit delayed startup (with or without additional bids to buy)
- (c) Other unit delayed shutdown (with or without additional bids to buy or sell)
- (d) Other unit startup (with or without additional bids to buy or sell)

- (e) Bid to buy or sell with brokerage
- (f) Attempt to buy or sell through bilateral transactions
- (g) Default on generation requirement with penalty incurred
- (h) Use available interruptible native load contracts
- (i) Combination of above

### 6.5.6 Efficiency Considerations

This subsection illustrates the scenarios when changes in commitment schedule could result in increased efficiency and savings. The changes are other than those identified by the commitment program, which only looks to schedule units for the forecasted generations requirement. This section explores the possibility of commitment change that is contingent upon a change in generation requirement because of potential sales or purchases.

#### 1. Avoiding shutting down an efficient unit during low load periods

This situation typically arises for participants with a low minimum load compared to their average load, and who own efficient units with a large  $P_{min}$ . If the participant could sell a sufficient amount of energy in the low load periods to keep the unit running, significant savings could be realized. The options available are:

- (a) Keep specified unit on (with bid to sell at low price)
- (b) Dump power with incurred penalty
- (c) Attempt to sell through bilateral transactions
- (d) Combination of above

#### 2. Avoiding starting an inefficient unit during peak load periods

This situation arises for utilities with a high peak load compared to average load,

and who own one or more inefficient units. For the peak hours, which may be as few as 3 or 4, expensive peaking units are started up. Avoiding these startups might result in significant savings. The options available are:

- (a) Keep specified unit off (with bid to buy at high price)
- (b) Default on generation requirement with penalty incurred
- (c) Attempt to buy through bilateral transactions
- (d) Combination of above

#### **6.5.7 Market Condition Considerations**

This subsection illustrates the cases where market conditions provide an opportunity for additional profits when commitment schedules are changed. The two cases considered are when market prices are unusually high and low.

##### **1. Market price forecast is high.**

In case of such a forecast, if a participant has a relatively low percentage of their generating capacity committed, then the following options are available:

- (a) Bid to sell as appropriate from currently committed or operating units only
- (b) Consider starting additional units and bid to sell
- (c) Attempt to sell through bilateral transactions
- (d) Combination from above

##### **2. Market price is forecast is low.**

In case of such a forecast, if the participant has a relatively high percentage of their generating capacity committed, then the following options are available:

- (a) Bid to buy as appropriate considering only decommitted or non-operating units only

- (b) Consider shutting down additional units and bid to buy
- (c) Attempt to buy through bilateral transactions
- (d) Combination of above

#### **6.5.8 Secondary-Effect Considerations**

The secondary effects on schedule changes, as described in Section 6.1, also result in one or more of the situations illustrated above, and will not be illustrated separately. Also, dependent on the resultant changes in unit status and transactions, the resultant net interchange in the hours affected may be more or less than initially planned.

### **6.6 Scheduling Considerations – Numerical Examples**

In this section, the concepts of utility using the Expo-Power utility function will be illustrated and contrasted with expected value maximization approach, for various scenarios where scheduling considerations come into play.

#### **6.6.1 Selling/Buying Without Changing Commitment Order**

Selling and buying without changing commitment order from a base case unit commitment was examined in Chapter 3. In this section, the approach is essentially similar to that approach, except that expected utility maximization is illustrated. However, in using the expected utility maximization approach in conjunction with the Expo-Power utility function, it must be mentioned that the choice of the parameters  $W$ ,  $\alpha$  and  $\beta$  affect the optimal solution. An increase in  $\beta$  unambiguously results in an increase in the participant's absolute risk aversion, in other words, the participant will choose a more conservative bid price with increasing  $\beta$ . The effect of  $W$  and  $\alpha$  are more complex. These are illustrated in Figures 6.4–6.6. In Figure 6.4, the expected utility from a sale scenario is illustrated. In the example shown, the selling participant models competitor behavior

in the form of an incomplete-beta function, and uses the Expo-Power objective function given in Equation 6.12. For this particular scenario, the parameters for optimization are as follows:

$c = 6.67$  \$/MWH (generating cost)

$\theta = 1$ ;  $\beta = 4.1667 \times 10^{-04}$  (utility function parameters)

$a = 15.7103$ ;  $b = 4.6644$ ;  $m = 15.0000$  (incomplete-beta function parameters)

Given these parameters, the ' $\phi$ ' on each curve represents the suboptimal,<sup>3</sup> normalized bid price  $x$  for various values of initial wealth,  $W$ , when  $\alpha = 1$ . As we can see, this value remains constant at 0.5315 \$/MWH. Thus, we can see that selecting  $\alpha = 1$  indeed does correspond to constant absolute risk aversion (CARA), and the participant would show the same degree of risk aversion, regardless of the level of initial wealth. Figure 6.5 shows the case where all other parameters are the same as before, except  $\alpha = 1.1$ . This corresponds to increasing absolute risk aversion with wealth (IARA). Though it is not easily apparent from the figure, the suboptimal bid price does shift to the left (lower values) for each higher-wealth curve, giving values of 0.4745, 0.4716, and 0.4693 respectively for initial wealths of 5000, 6000, and 7000. Since we are considering a seller, this does mean that the seller becomes more "conservative" in its bidding strategy with increasing wealth, for a constant bid size. This may not be common in real life. But it certainly can be modeled if the situation arises, by simply selecting a value for  $\alpha > 1$ . Figure 6.6 shows the case of decreasing absolute risk aversion with wealth (DARA), which is the most conventional assumption of risk attitudes in commodity trading models. Here,  $\alpha = 0.8$ . For this case, the suboptimal values increase with increasing wealth, resulting in values of 0.5607, 0.5620, and 0.5629 respectively for initial wealths of 5000, 6000, and 7000. Thus, we can see that the seller bids a more "risky" bid price with an increase in wealth, for a constant bid size.

---

<sup>3</sup>Rather than implement customized code for each such scenario, the results shown in this chapter were generated using Matlab Version 5.0, and its Optimization Toolbox. The specific routines used were *fmin* for unconstrained maximization, and *constr* for constrained maximization.



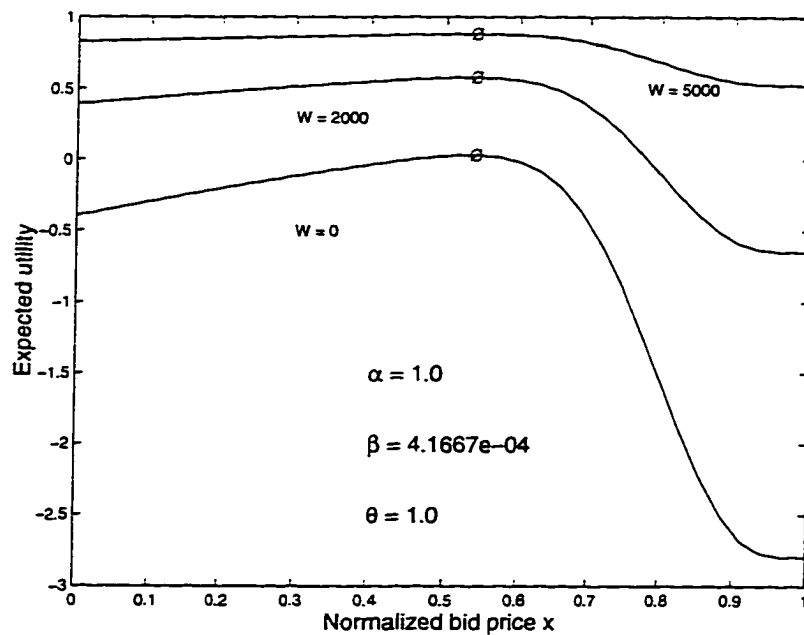


Figure 6.4 Variation of optimal solution with wealth: CARA.

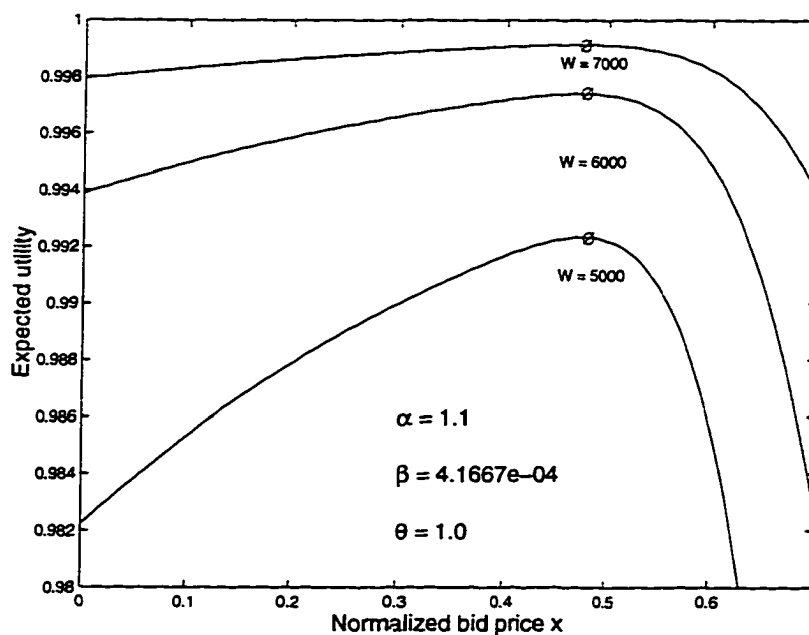


Figure 6.5 Variation of optimal solution with wealth: IARA.

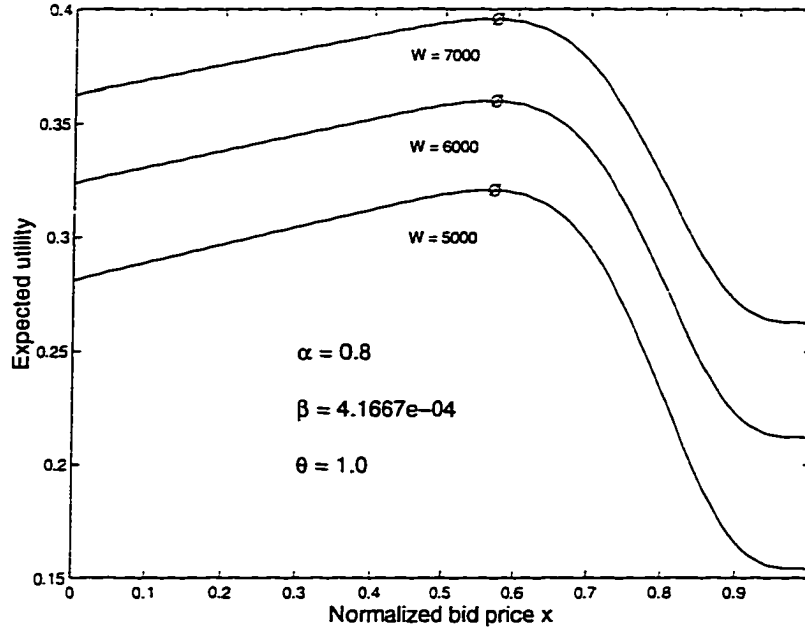


Figure 6.6 Variation of optimal solution with wealth: DARA.

The proposed expected utility model results in interesting changes in suboptimal bids for different parameter values, even for the simple case of static commitment order. In the following sections, we attempt to model some of the complexities involved in including schedule changes in the objective function.

### 6.6.2 Avoiding Startups/Shutdowns

In this section three situations will be analyzed that could involve a participant considering a change in the commitment schedule of generating units. In order to do this, two approaches will be presented for each of the different scenarios. One is an expected profit maximization approach, and the other is an expected utility maximization approach. The expected profit maximization approach has been illustrated in reasonable detail in previous chapters.

### 6.6.2.1 One-Hour Purchase with Avoided Startup

The first scenario involves a small system of four generating units, where the purchase of energy during one hour could result in avoiding the startup of an inefficient peaking unit, with significant savings. Let us consider a four generating unit system that has a base case unit commitment for a period of 168 hours as shown in Figure 6.7. The dashed curve represents system generating requirement plus reserves. The curve above that is the online capacity curve. It can be seen that in certain periods, a generating unit needs to be started up to meet spinning reserve requirement. Upon closer examination of the generating unit characteristics (not shown), it was observed that one of the peaking units was started up at hour 42, and was on until hour 48, when it was shut down. This is because the unit's minimum up time is six hours. Also, upon examination of the load curve, it was observed that a purchase of 20 MW during hour 42 would be sufficient to avoid this startup and shutdown, thus resulting in savings. In order to determine the magnitude of savings, a second unit commitment was performed, with the load for hour 42 reduced by 20 MW. This unit commitment is shown in Figure 6.8. The difference between the generating costs resulting from the two commitments was determined to be  $\$275,040 - \$271,810 = \$3230$ . Thus, the value of the purchase of 20 MW for one hour was determined as  $\$3230 / (20 \times 1) = 161.5 \text{ \$/MWH}$ . This value will now be used to analyze the bidding strategy of the participant, from both an expected profit maximization and an expected utility maximization point of view.

The bid distribution parameters for the participant will be assumed to be as follows:

Beta distribution with

$$a = 15.7103$$

$$b = 4.6644$$

$$m = 15.0000$$

The Expo-Power utility function parameters will be assumed to be:

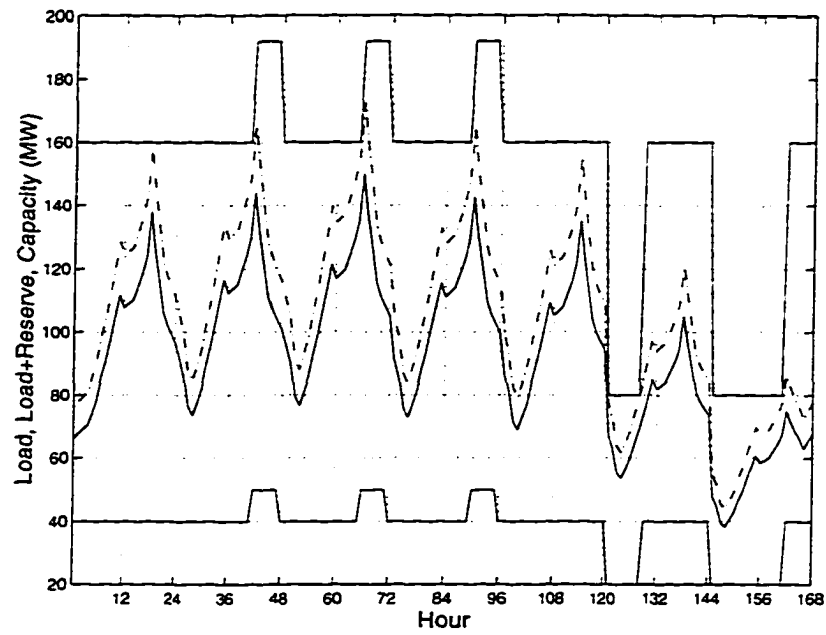


Figure 6.7 Base case unit commitment for 4-unit system.

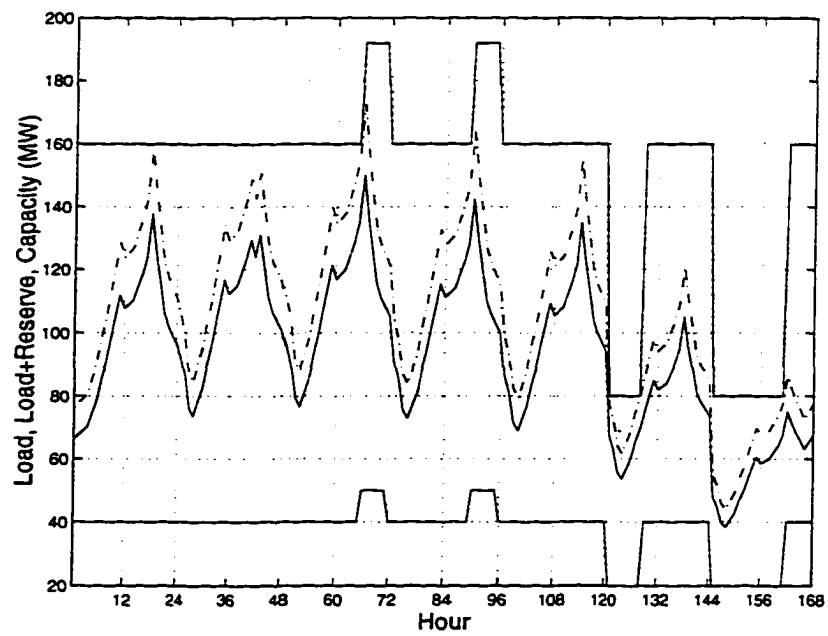


Figure 6.8 Purchase case unit commitment for 4-unit system.

$$\alpha = 1$$

$$\beta = 4.1667 \times 10^{-4}$$

$$\theta = 1$$

$$W = 0$$

Also, cost of generation  $c = 161.5$  \$/MWH. For this case, the expected value maximization result is obtained by maximizing the following objective:

$$F(x, a, b)(c - mx) \quad (6.13)$$

where  $F(x, a, b)$  is the incomplete-beta function, and represents the probability of success of bid  $x$ ,  $S(x)$ . The expected utility maximization result is obtained by maximizing the following objective:

$$F(x, a, b) \left( \theta - e^{-\beta(W + 20(c - mx))^\alpha} \right) \quad (6.14)$$

The optimization results are given in Table 6.4. EV-Max indicates expected value maximization, and EU-Max indicates expected utility maximization. It can be seen that the expected value maximization solution has a lower bid price ( $x^*$  and  $p_b^*$ ) than the expected utility maximization solution. This is consistent with the fact that the latter solution takes into effect the risk aversion of the buyer. Consequently, the probability of acceptance ( $S(x^*)$ ), and expected utility ( $EU_b(p_b^*)$ ) of the suboptimal bid  $x^*$  are higher for the latter case.

An interesting point to be noted is regarding  $c$ , which is very large for current day fuel prices. This indicates that there may be significant cost savings for a participant, if a longer term contract were entered into, for purchase of peak capacity and energy. The effect of such a contract on bidding strategies would be a lower  $c$ , which will lead to a lower  $x^*$ . In the next two examples, we examine cases of where  $c$  is much lower, and falls within a reasonable price range.

Table 6.4 Variation of suboptimal values of 4-unit system with strategy.

Strategy	$x^*$	$p_b^*$	$S(p_b)^*$	$E_{lb}(p_b^*)$	$EU_{lb}(p_b^*)$
EV-Max	0.9665	14.4975	0.9992	146.88	0.9946
EU-Max	0.9927	14.8903	0.9999	146.61	0.9954

#### 6.6.2.2 11-Hour Purchase with Avoided Startups

The following two scenarios involve the analysis of purchase and sale of power for a larger 16-unit system, over a period of time longer than one hour. The analysis presented is meant to be an illustration of some of the considerations that could go into the decision of whether to bid or not, and if so, what bid price to select.

Let us now consider a 16-unit system that has a base case unit commitment for a period of 168 hours as shown in Figure 6.9. Upon examination of the load curve, it was observed that a purchase of 200 MW during hours 14 through 24 would be required to avoid some startups and shutdowns, thus resulting in savings. In order to determine the magnitude of savings, a second unit commitment was performed, with the load for hours 14 through 24 reduced by 200 MW. This unit commitment is shown in Figure 6.10. The difference between the generating costs resulting from the two commitments was determined to be \$2,396,200 - \$2,383,400 = \$12,800. Thus, the value of the purchase of 200 MW for 11 hours was determined as  $\$12,800 / (200 \times 11) = 5.8182 \text{ \$/MWH}$ . This value will now be used to analyze the bidding strategy of the participant.

The same bid distribution and utility function parameters will be assumed again for this participant. For the first hour of the proposed purchase bid, the modeling is identical to that for the previous case. However, once the bid has been accepted, and the purchase schedule is committed to, the participant has to consider the possibility of the subsequent hours' bids being rejected. Unlike in the previous cases, there is a monetary impact of a rejected bid for this situation. The impact could be modeled in

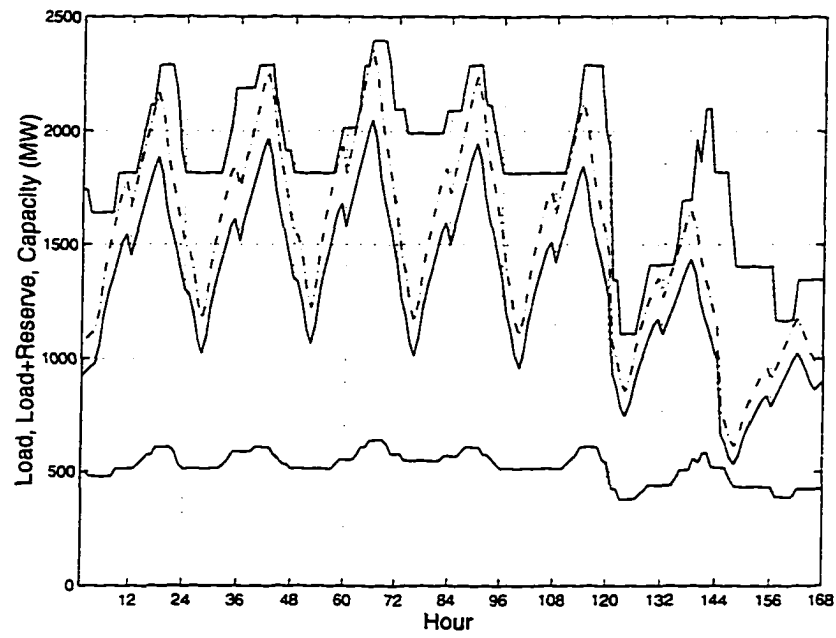


Figure 6.9 Base case unit commitment for 16-unit system.

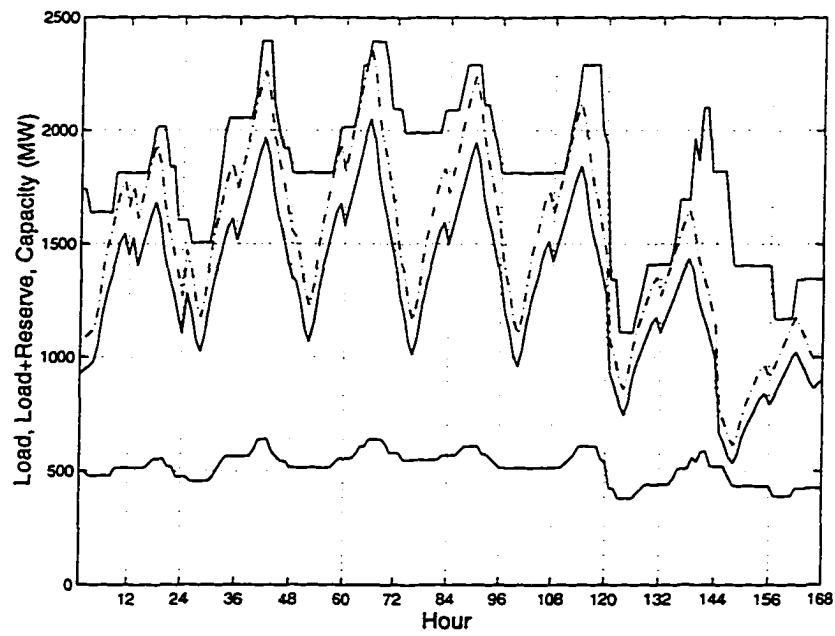


Figure 6.10 Purchase case unit commitment for 16-unit system.

number of detailed ways including a decision tree analyzing the possible actions of the participant. However, to keep the modeling clear conceptually, we choose to model this “downside” risk simply by a constant penalty,  $p$  \$/MWH. The justification for this is that in the worst case, a rejected bid can be replaced by energy from one of a number of alternative sources, such as peaking units, dispatchable contracts or even emergency power contracts.  $p$  simply represents the cost of using one of these alternatives<sup>4</sup>. A conservative participant might use the most expensive of these sources to model the downside risk. Adding this component into the model, the expected value maximization and expected utility maximization objectives become:

$$F(x, a, b)(c - mx) + (1 - F(x, a, b))(-p) \quad (6.15)$$

$$F(x, a, b) \left( \theta - e^{-\beta(W+200(c-mx))^\alpha} \right) + (1 - F(x, a, b)) \left( \theta - e^{-\beta(W-200p)^\alpha} \right) \quad (6.16)$$

For the first hour of the proposed purchase, the value for  $p = 0$ , because if the bid is rejected, the participant can choose to stay with the original commitment schedule. For subsequent hours, let us assume that  $p = 20$ . Then, Table 6.5 shows the values for suboptimal bid prices for the first hour and the subsequent hours. Since the penalty for the first hour is different from the penalty in the subsequent hours, two different sets of solutions are obtained.

Table 6.5 Variation of suboptimal values of 16-unit system with strategy (purchase case).

Strategy	$x^*$	$p_b^*$	$S(p_b)^*$	$E_{lb}(p_b^*)$	$EU_{lb}(p_b^*)$
EV-Max, 1st hour	0.3617	5.4260	6.8802E-5	2.6981E-5	2.2121E-6
EU-Max, 1st hour	0.3621	5.4318	6.9825E-5	2.6978E-5	2.2124E-6
EV-Max, other hours	0.9196	13.7938	0.9733	-8.2963	-1.0331
EU-Max, other hours	0.9341	14.0114	0.9874	-8.3424	-1.0212

<sup>4</sup>Also conceivable, is a  $p$  that varies with the amount of time that is left in the transaction, or one that is dependent on how much wealth has been recovered in the past hours. However, these are more complex to model, and we assume a constant  $p$  for this scenario.



It may be seen that once again, expected utility maximization leads to a more conservative (higher) buy bid than expected value maximization. Also, it may be seen that this effect is more pronounced for the subsequent hours case, when there is a non-zero downside. In other words, when the downside risk is significantly large, the participant's risk aversion is observed to have a larger effect. Another interesting observation is that the maximum expected utility in the case where  $p = 20$ , is negative. This is an indication to the participant that the purchase bid is not an advisable activity. This is also consistent with the fact that  $c = 5.8182$  is a relatively low value of generation, given the bid distribution parameters. In other words, for this purchase bid, the participant's cost curve is not competitive enough to justify the risks. Thus, we can see that the results from the suboptimal bidding strategy predict a situation where the participant should not submit a buy bid.

#### 6.6.2.3 15-Hour Sale with Avoided Shutdown

Let us now consider the 16-unit system again, from the point of view of a seller. From the load it was observed that a sale of 120 MW during hours 20 through 34 would be required to avoid the shutdown of a base load unit during off-peak hours, thus resulting in savings. In order to determine the magnitude of savings, a second unit commitment was performed, with the load for hours 20 through 34 increased by 120 MW. This unit commitment is shown in Figure 6.11. The difference between the generating costs resulting from the two commitments was determined to be  $\$2,408,200 - \$2,396,200 = \$12,000$ . Thus, the additional cost of the sale of 120 MW for 15 hours was determined to be  $c = 12,000 / (120 * 15) = 6.66$  \$/MWH. This value will now be used to analyze the bidding strategy of the participant.

Again, the same bid distribution and utility function parameters will be used. Also, a penalty  $p = 20$  will be assessed for a rejected bid, in hours subsequent to the first hour of the proposed sale. This penalty is a representation of alternatives the participant

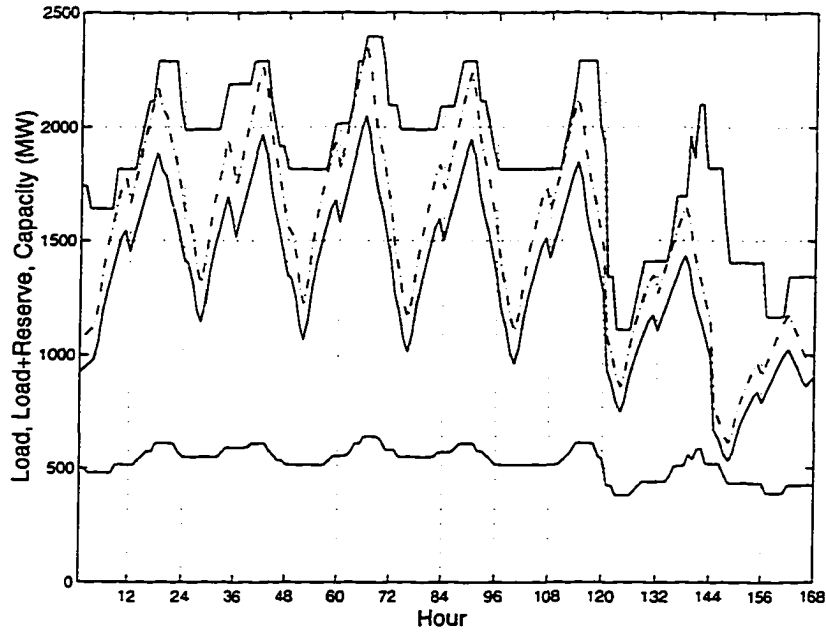


Figure 6.11 Sale case unit commitment for 16-unit system.

might have to satisfy generation requirement constraints, such as shutting down units, dispatchable sale contracts, pumped storage units, or even dumping power into the system, each with its associated cost. The objectives for expected value maximization and expected utility maximization become:

$$(1 - F(x, a, b))(mx - c) + F(x, a, b)(-p) \quad (6.17)$$

$$(1 - F(x, a, b)) \left( \theta - e^{-\beta(W + 120(mx - c))^\alpha} \right) + F(x, a, b) \left( \theta - e^{-\beta(W - 120p)^\alpha} \right) \quad (6.18)$$

Results of optimization are given in Table 6.6. It may be observed that expected utility maximization bid prices are lower (more conservative from the seller's point of view) than expected value maximization prices. Also, this effect is more pronounced for the case where the penalty is non-zero. Thus, depending on if the participant chooses to use expected value maximization, or expected utility maximization, i.e., depending on whether the participant is risk neutral, or risk averse, the analysis presented shows what the suboptimal bid price should be.

Table 6.6 Variation of suboptimal values of 16-unit system with strategy (sale case).

Strategy	$x^*$	$p_s^*$	$S(p_s)^*$	$E_{lb}(p_s^*)$	$E\tilde{U}_{lb}(p_s^*)$
EV-Max, 1st hour	0.7077	10.6154	0.7676	3.0311	0.1374
EU-Max, 1st hour	0.7016	10.5241	0.7849	3.0251	0.1376
EV-Max, other hours	0.5864	8.7964	0.9661	1.3766	0.0392
EU-Max, other hours	0.5545	8.3174	0.9823	1.2638	0.0472

The scenarios just presented are examples of buying and selling situations with a commitment change being considered. They are by no means an exhaustive list of such scenarios. However, the purpose of illustrating the complexities involved in considering commitment changes in bidding decisions is achieved by these scenarios.

### 6.6.3 Simultaneous Buy/Sell Activity

The previous section considered cases where the participant had already made a decision to bid for either buying or selling at a given hour. In this section, we consider the situation where the participant bids to buy and sell simultaneously. In this situation, let us assume that the participant has, by means similar to the above section, determined a cost/value of the energy for both the purchase and the sale block of energy. Let these costs be given by  $c_b$  and  $c_s$  respectively. Then, three possible cases are of interest. These cases arise out of three possible comprehensive production cost shapes, shown in Figure 6.12.

In the first case, the operating point is such that  $c_b < c_s$ . In the other two cases,  $c_b > c_s$ , with the difference being greater in the last case. The following analyses illustrate the application of the expected value and expected utility maximizing approaches to these three cases.

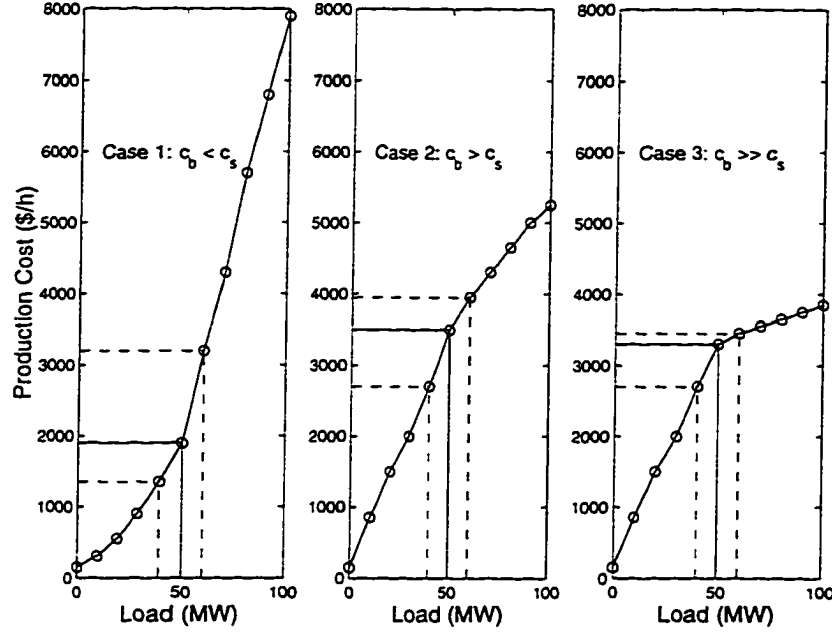


Figure 6.12 Simultaneous buy/sell: three possible cases.

#### 6.6.3.1 Case 1 where $c_b < c_s$

Let us assume that  $c_b = 8.99$  \$/MWH, and  $c_s = 9.04$  \$/MWH, for a block of energy, say 200 MW. All other parameters for bid distributions and utility function remain the same. Further, let us assume that the participant considers both buy and sell bids to be distributed according to the same distribution<sup>5</sup>. Then, the objectives for expected value and expected utility maximization models are as follows:

$$(1 - F(x_s, a, b))(mx_s - c_s) + F(x_b, a, b)(c_b - mx_b) \quad (6.19)$$

$$(1 - F(x_s, a, b)) \left( \theta - e^{-\beta(W + 200(mx_s - c_s))^\alpha} \right) + F(x_b, a, b) \left( \theta - e^{-\beta(W + 200(c_b - mx_b))^\alpha} \right) \quad (6.20)$$

where  $x_b$ ,  $x_s$  are the normalized buy and sell bid prices respectively. It can be observed for this case, that the objective function is separable into two separate cases, sell and buy. Suboptimal solutions are given in Table 6.7. The usual observations can be made

<sup>5</sup>None of these assumptions are restrictive from the point of view of modeling: they are made primarily for the sake of simplicity.

regarding the effect of risk aversion on the suboptimal values. Again, depending on the risk attitudes of the participant, the analysis shows different solutions to the bidding problem, with the expected utility maximization strategy being the more “conservative” of the two strategies. This means that the sell bid price is lower, and the buy bid price is higher than in the expected value maximization strategy.

Table 6.7 Variation of suboptimal values for simultaneous buy/sell – Case 1.

Strategy	$x_b^*$	$x_s^*$	$p_b^*$	$p_s^*$	$E_{lb}(p_b^*, p_s^*)$	$EU_{lb}(p_b^*, p_s^*)$
EV-Max	0.5527	0.7551	8.2898	11.3270	1.3920	0.1057
EU-Max	0.5638	0.7499	8.3069	11.2480	1.3897	0.1058

#### 6.6.3.2 Case 2 where $c_b > c_s$

Let us assume that  $c_b = 10.99$  \$/MWH, and  $c_s = 9.04$  \$/MWH. Then the objectives are the same as before, and results of optimization are shown in Table 6.8. The effect of modeling risk aversion through the utility function, is again an increase in the suboptimal buy bid price and a decrease in the suboptimal sell bid price. Depending on the risk attitudes of the participant, the analysis shows different solutions to the bidding problem.

Table 6.8 Variation of suboptimal values for simultaneous buy/sell – Case 2.

Strategy	$x_b^*$	$x_s^*$	$p_b^*$	$p_s^*$	$E_{lb}(p_b^*, p_s^*)$	$EU_{lb}(p_b^*, p_s^*)$
EV-Max	0.6653	0.7551	9.9795	11.3270	1.5114	0.1152
EU-Max	0.6675	0.7499	10.0126	11.2484	1.5092	0.1154

#### 6.6.3.3 Case 3 where $c_b \gg c_s$

Let us assume that  $c_b = 12.99$  \$/MWH, and  $c_s = 8.04$  \$/MWH. From the modeling point of view, this situation poses an interesting problem. Since the broker matches highest bidder for buying to lowest bidder for selling, there is a higher probability in this case, that the participant might be matched to sell to itself. In the event of such

a match, the savings are not real. To avoid this situation, an additional constraint has to be imposed, and that is  $x_b \leq x_s$ . This will prevent the broker from matching the participant's buy bid to its own sell bid. However, the more profitable buy bid situation (more profitable because  $c_b \gg c_s$ ) is artificially limited by this constraint. Thus, we expect the optimal solution to be at the boundary, where is  $x_b = x_s$ . Table 6.9 shows that this is indeed the case. Further, for the expected utility maximization model with constraint imposed, the suboptimal bid prices are lower than those for the expected value maximization model with constraint imposed. Thus, once again risk aversion effects are illustrated, with the additional constraint imposed if the participant wishes to avoid the situation where there is a match between two of his own bids.

Table 6.9 Variation of suboptimal values for simultaneous buy/sell – Case 3.

Strategy	$x_b^*$	$x_s^*$	$p_b^*$	$p_s^*$	$E_{lb}(p_b^*, p_s^*)$	$EU_{lb}(p_b^*, p_s^*)$
EV-Max (no addl. constr.)	0.7619	0.7326	11.4290	10.9885	2.6895	0.2015
EU-Max (no addl. constr.)	0.7660	0.7254	11.4898	10.8803	2.6843	0.2019
EV-Max (with addl. constr.)	0.7422	0.7422	11.1336	11.1336	2.6641	0.1981
EU-Max (with addl. constr.)	0.7393	0.7393	11.0899	11.0899	2.6631	0.1982

#### 6.6.4 Consideration of Take-or-Pay Fuel Contract Requirements

In this section, a fuel contract with a take-or-pay fuel block is included in the bidding considerations. Let us assume that the selling participant is considering a take-or-pay fuel contract that can be consumed by dispatching a certain mix of units at 100 MW. Further, let us assume that the cost of this contract, if used by this mix of units, works out to be 8 \$/MWH. The participant now considers satisfying take-or-pay requirements by bidding to sell 100 MW in the brokerage. For this situation,  $c = 8$ , and the penalty for a rejected bid is simply the cost of the take-or-pay contract itself. So we can model

it as a 8 \$/MWH penalty. This problem then reduces to the earlier case of considering a penalty for rejected bid, with  $p = 8$ . The objective functions are similar to the ones given in Equations 6.17 and 6.18.

Results of optimization are given in Table 6.10, with the usual implications. If the participant thus wishes to satisfy take-or-pay requirements by bidding to sell through the brokerage, then the bid prices for different risk attitudes should be as shown in Table 6.10.

Table 6.10 Variation of suboptimal values for take-or-pay fuel block consideration.

Strategy	$x_s^*$	$p_s^*$	$S(p_s)^*$	$E_{lb}(p_s^*)$	$EU_{lb}(p_s^*)$
EV-Max	0.6400	9.6000	0.9122	0.7574	0.0241
EU-Max	0.6275	9.4126	0.9287	0.7411	0.0249

### 6.6.5 System Health Considerations

Previously, the risk that we have included in modeling has been price risk. Production risk is the other kind of risk that exists, and will be included in the model as follows. First, the assumption is made that the two kinds of risks are independent. In other words, the participant's generating system health, for example, has no effect on market prices. This assumption is not always satisfied. For example, if the participant is a large generating company owning a significant percentage of a region's generating capacity, then market prices and bids could be affected by perceived outage risks in that participant's generation mix. A typical example would be the northeastern region of the United States at the time of writing this dissertation, where certain large nuclear units are under risk of outage periodically. This situation does have a tangible effect on the market behavior of participants of the energy markets in that region. However, for a number of situations the independence requirement is satisfied. In this section, we will model a unit outage possibility as part of the bidding decision.

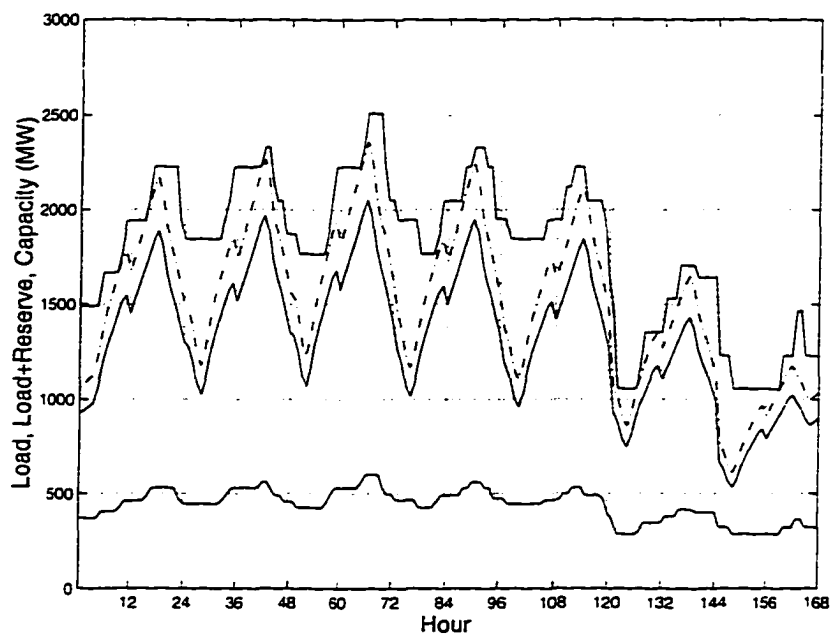


Figure 6.13 Base case unit commitment for 16-unit system (with unit outage).

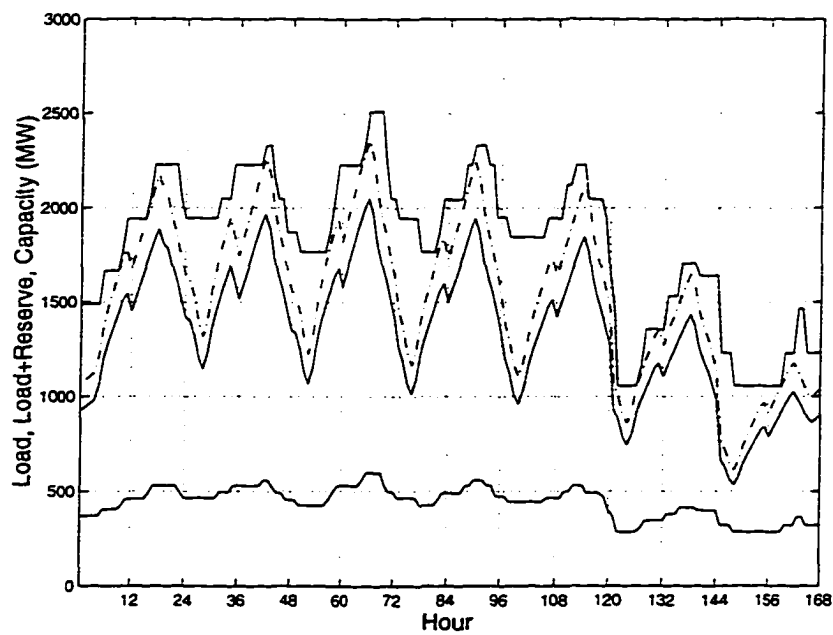


Figure 6.14 Sale case unit commitment for 16-unit system (with unit outage).



Let us consider again, the 16-unit system of Section 6.6.2.3, for the identical sale of 120 MW over a period of 15 hours, in hours 20 through 34. Now, we add the possibility that the outage of one of the generating units, with a forced outage rate (f.o.r.) of 4.1 %, i.e., with a probability of forced outage,  $p_{f.o.r}$  of 0.041, is to be modeled. In other words, after the participant has agreed to the sale, this unit could experience an outage. That could have a deleterious effect on the profitability if the sale. To model this, we perform two more unit commitment simulations. The first is a base case simulation with the unit unavailable, and a sale case with the 15 hour sale in place, with the unit unavailable. The results from simulation are shown in Figures 6.13 and 6.14 respectively.

The price of the sale from these simulations is calculated in a manner similar to Section 6.6.2.3 as  $\hat{c} = 10.11$  \$/MWH. Thus, the loss of the unit results in an increase in the costs from  $c = 6.67$  \$/MWH. Now, given that the probability of forced outage, and the bid distributions are independent, the following objectives can be derived for the bidding problem. The objective for the expected value maximizing participant is as follows:

$$(1 - p_{f.o.r.}) \{ (1 - F(x, a, b))(mx - c) + F(x, a, b)(-p) \} + p_{f.o.r.} \{ (1 - F(x, a, b))(mx - \hat{c}) + F(x, a, b)(-p) \} \quad (6.21)$$

The objective for the expected utility maximizing participant is as follows:

$$(1 - p_{f.o.r.}) \left\{ (1 - F(x, a, b)) \left( \theta - e^{-\beta(W + 120(mx - c))^\alpha} \right) + F(x, a, b) \left( \theta - e^{-\beta(W - 120p)^\alpha} \right) \right\} + p_{f.o.r.} \left\{ (1 - F(x, a, b)) \left( \theta - e^{-\beta(W + 120(mx - \hat{c}))^\alpha} \right) + F(x, a, b) \left( \theta - e^{-\beta(W - 120p)^\alpha} \right) \right\} \quad (6.22)$$

Results of optimization are given in Table 6.11. Upon comparison with Table 6.6 shows that all the suboptimal bid prices are higher for the case where system outage risk is included. In the expected value maximization case, the possibility of outage has made

the participant effectively revise the generating cost as a probability weighted average of healthy and outaged costs. In the expected utility maximization case, a similar effect has occurred, with the added consideration of price risk aversion. Admittedly, such an inclusion on system health risk is only as good as the forced outage probability is. However, if the participant wishes to include a more detailed model for incorporating outage risk, the bidding model can certainly be modified to include it.

Table 6.11 Variation of suboptimal values of 16-unit system with strategy (sale case considering unit outage possibility).

Strategy	$x^*$	$p_s^*$	$S(p_s)^*$	$E_{lb}(p_s^*)$	$EU_{lb}(p_s^*)$
EV-Max, 1st hour	0.7100	10.6504	0.7607	2.9208	0.1325
EU-Max, 1st hour	0.7043	10.5648	0.7773	2.9178	0.1326
EV-Max, other hours	0.5868	8.8018	0.9659	1.2404	0.0323
EU-Max, other hours	0.5551	8.3261	0.9821	1.1291	0.0403

## 7 SUGGESTIONS FOR FUTURE WORK AND CONCLUSIONS

### 7.1 Suggestions for Future Work

The research presented in this dissertation was a first-attempt at developing a bidding strategy framework for the energy brokerage market. While progress was made towards this goal, it is recognized that many issues remain unresolved, and many problems unsolved in this area. Of these, the following areas are suggested as areas for further research.

#### 7.1.1 Market-Based Strategies

- The market rules used in this research were very simple, in order not to distract from the stated goal of strategy development. However, given the fact that several complex markets are evolving currently, such as the California ISO-PX market, it would be of value to extend the strategies developed here to fit a single clearing-price model, in addition to the bilateral market structure assumed here.
- The lower bounding result derived and used in this research lead to the suboptimality of the bidding solution. This lower bounding was needed primarily because of the extremely limited form of information that was assumed as being public. Exploring the effects of assuming that more data is publicly available could provide a basis for further research. This would involve the determination of how each new

data item would impact the strategies and their outcomes.

- Transmission is modeled at a rudimentary level in this research. An enhanced transmission model, with the inclusion of transmission loss-allocation, and ancillary service considerations would provide a more realistic representation of the energy markets. This could involve using estimates of anticipated line loadings, transmission usage costs, and ancillary service costs. The mechanisms for obtaining and modeling these estimates, and the impacts of such detailed models on the effectiveness of bidding strategies is a good candidate for further research.
- From a modeling perspective, a detailed survey of different probability distributions in addition to the ones modeled in this research, would be of value. Enhancing the simulator to add an automatic regression feature would greatly reduce the time spent in evaluating such distributions. Currently, regression is performed by manually interfacing the data with a statistical package such as SPSS.
- Analytical sensitivities of the results from suboptimal bidding to the estimated parameters of competitors' bid distributions may not be possible for the models proposed here. However, a detailed study of numerical sensitivities would be a useful area to research further. Such a sensitivity study would also provide better ways to design and implement heuristics for tuning of the suboptimal bids.
- The initial testing of the strategies presented in Chapter 5 involved the assumption that only one participant uses the strategy at a time. This procedure was used to test the effect of a strategy when it is assumed that market conditions do not change substantially from the time the market data was obtained to the time actual bidding for the subsequent periods is performed. Variations on this format of testing would help identify the true value of these strategies under different market conditions. For this, several rounds of simulation, with each participant

using various strategies based upon the results of the previous round, would provide more insights. Again, for this kind of testing to be implemented on a large scale, inclusion of an automatic regression feature in the simulator would be of value.

### 7.1.2 Scheduling-Based Strategies

- The illustrative scenarios described in this research are somewhat limited by the capabilities of the simple unit commitment program used. It would be of value to interface the simulator to a more versatile unit commitment or scheduling package in order to further explore the effects of changes in commitment schedules.
- In some of the scenarios presented in Chapter 6, the penalties for defaulting on a transaction were assumed to be simple numbers. Further research in this area could include the explicit modeling of stand-by units, pumped-storage hydro units, emergency dispatchable generation contracts, or insurance and financial contracts, to determine a realistic value for this penalty.
- The use of utility functions to model risk assumed that the participants already knew what parameters to select. However, the selection of the appropriate utility function parameters is in itself an interesting and challenging area for future research.
- The availability of financial tools for risk management currently is somewhat limited. But in future, if such tools evolve into more widely used and effective means for hedging risk, then the objective functions in the bidding process could change. Exploration of possible objective functions is another possible area for future research.
- In Table 1.1, several factors were listed, that could affect the bidding strategies of the participant. Of these, only a limited number have been discussed within the

scope of this research. Inclusion of some of these factors into bidding strategies would be a natural extension of this work. Examples include the inclusion of unit maintenance requirements, load-related factors, fuel-related factors and emission-related factors, into the bidding strategies.

## 7.2 Conclusions

In this dissertation, a framework has been presented for the development of bidding strategies for the individual participants in an energy brokerage. The framework included the definition of different types of participants, the factors that affected the bidding behavior of the participants, and a probabilistic model to incorporate available market information in the determination of bid prices for the participants. Based on this probabilistic approach, various distributions were selected for modeling competitor bids, and expressions were derived for the determination of optimal bid prices. A simulator was developed to implement and test these strategies. The following section presents the conclusions reached from this activity.

### 7.2.1 Market-Based Strategies

The probabilistic modeling of competing bids has been shown to hold promise, although future research is expected to enhance and improve the effectiveness of this approach. The three functions selected for modeling the competitors' bidding behavior in this research were the polynomial, incomplete-beta and incomplete-gamma functions. The relative advantages and disadvantages of these distributions were discussed in Chapter 3. Overall, the incomplete-beta function approach seems to be the most flexible of the three distributions. The polynomial distribution has the advantage of being simple to obtain a fit for, as well as to implement. But it suffers from the accuracy point of view in satisfying the CDF property of being bounded by 1. Also, the multiple solutions

given by this model are not guaranteed to be real numbers. Furthermore, heuristics are difficult to implement for this distribution. The incomplete-gamma function is very similar to the incomplete-beta function in its properties. However, because of the fact that its domain is not finite (normalized), it is difficult, using this distribution, to obtain a proper fit to market data. Thus, the incomplete-beta function approach was found to be the best of the three because it is flexible, and relatively easy to implement model.

Based on the simulations performed on the test system, shown in Chapter 5, the following conclusions are drawn.

- The availability of market information, such as transaction prices, can be used to model competitors' bidding behavior. The simulations showed that, while there is promise in using the limited information to improve bidding performance, transaction prices are not always an effective proxy for actual bid information.
- An attempt was made in this research, to introduce heuristic tuning of the bids by the participants, after the probabilistic method was completed. The results clearly do not indicate which of the heuristics is superior. However, the results do indicate that, in the case of the incomplete-beta function, it is possible to improve the performance by more detailed modeling of the competitors.
- Also, because of the presence of the normalizing bid-price factor  $m$ , it is possible to add a certain amount of "intelligence" to the bidding strategy, by incorporating a market spread in the parameters of the distribution. Exactly how participants might arrive at this spread was not explored in this research: only how to incorporate such information was discussed. However, the results indicate how participants would be able to improve their performances, *if* such conjectures about the spread in the market were to become available to them.
- The extent to which participants would be willing to model their competitors could

depend on their own costs, objectives, as well as on their key competitors. For example, Company 1 and Company 5 in the test system were the predominant sellers in the simulations presented in Chapter 5. Also, when one of these companies attempted to increase their sell bid prices beyond a certain point, it was observed that the *other* company “cornered” the market. This suggests that the two companies have similar cost and capacity structures. (Examination of the production cost databases of the two companies shows that indeed, this is the case. But access to each other’s production cost databases is not necessary for the companies to *conjecture* that this is the case.) This could be sufficient motivation for the companies to model each other’s bidding behavior in more detail, or, if this is not possible, to force their bidding strategy to be more conservative. In addition, such advantages could motivate some new companies to be created, that provide market intelligence services to the participants.

- Transmission impacts the effectiveness of the strategies in a complex manner. Two types of effects were observed from the simulations. One was the rejection of transactions because of violations in transmission line limits. The second was the rejection of transactions because the energy cost savings was insufficient to justify the incurred transmission costs. Of these, the former is a factor that will have to be considered regardless of the rules of the market. In other words, system security constraints will always directly affect the effectiveness of a strategy, if the proposed transactions violate them. The latter constraint is a result of our assumption that the broker takes transmission usage costs into account during the matching process. In some of the evolving markets, it is left to the participants to reserve transmission usage independently of the energy market. In this case, transmission usage costs will have to be incorporated into the cost components of the bidding strategies. In all likelihood, this will make some transactions non-cost-effective.



In this research, we have not attempted to do this. The simulations included only indicate the extent to which such considerations affect the performance of the participants, and the simulations indicate that the effects could be significant. However, it is reasonable to expect that participants should develop methods that will incorporate transmission usage costs into the bidding process.

### 7.2.2 Scheduling-Based Strategies

In Chapter 6, an attempt was made to illustrate how the strategies of a company that owns and operates generating facilities would be affected by scheduling considerations. Some operating scenarios were described, which illustrated the complexities that are added to the strategic bidding problem when various unit commitment factors are included. It was also recognized that uncertainties in scheduling introduces risk. The use of utility functions was suggested as a means to model the attitudes of the participants towards this risk. Results were presented that contrast the expected utility maximization approach with the expected profit maximization approach followed in Chapter 3.

The Expo-power utility function developed by Saha [4] was selected for this modeling because of its flexibility in modeling various types of risk aversion. An additional parameter  $W$  was introduced to take into account the initial wealth of the participant. The comparisons, between the expected utility and expected profit maximization approaches, indicate that the utility function approach provides a mechanism to include risk considerations in strategic bidding in a consistent manner. However, it must be emphasized that the accuracy of this model in representing a participant's risk attitudes depends on how well the participants select the parameters of the utility function.

Generating system health considerations are very complex, and to include these in a detailed manner in bidding strategies is beyond the scope of this research. But an attempt was made to illustrate the complexities involved by considering the simple case of a unit outage, modeled through a unit forced outage rate. The example shows the

effect of such an outage on the suboptimal bid prices. However, the extent to which participants wish to model generating system health risks in their bidding strategies depends on a number of factors, including the health of the system, the resources available to the participant, and alternate mechanisms that the participant may choose to spread the risk, such as insurance, or financial hedging tools.

In conclusion, this research has provided a framework for strategy development for bidding in an energy brokerage. A clear definition of the problems associated with strategic bidding has been presented. Initial attempts at providing solutions to the problems, while not being conclusively successful, have opened up several promising avenues for further research. These avenues include, but are not limited to, the application of probabilistic modeling of competitor bids, the application of heuristic tuning, and the development and testing of such heuristics with the aid of a simulator. An attempt has also been made to incorporate the risk preference attitudes of participants, into the strategic bidding process. The use of utility functions to achieve this purpose has been successfully demonstrated. It is expected that the results of this research will not only be directly applicable to the electric energy market industry, but also to future researchers in this area.

## APPENDIX

In this chapter, the derivation of first order and concavity conditions for optimality are presented for the various distributions modeled in Chapter 3.

### A.1 Derivation of First Order Conditions for Optimality

#### A.1.1 Polynomial Modeling

Consider the objective function for a buyer given in Equation 3.16, reproduced below:

$$E_{lb}(p_b) = (c - p_b)(a_0 + a_1p_b + a_2p_b^2 + \dots + a_{n-1}p_b^{n-1} + a_np_b^n) \quad (\text{A.1})$$

For first order conditions to be satisfied, the first derivative of  $E_{lb}(p_b)$  should be equal to zero. In other words

$$\begin{aligned} E'_{lb}(p_b) &= 0 \\ \Rightarrow & -(a_0 + a_1p_b + a_2p_b^2 + \dots + a_{n-1}p_b^{n-1} + a_np_b^n) \\ & + (c - p_b)(a_1 + 2a_2p_b + \dots + (n-1)a_{n-1}p_b^{n-2} + na_np_b^{n-1}) = 0 \end{aligned} \quad (\text{A.2})$$

Simplifying the above equation, we get:

$$\begin{aligned} 0 &= -a_0 - a_1p_b - \dots - a_{n-1}p_b^{n-1} - a_np_b^n \\ &+ ca_1 + (2ca_2 - a_1)p_b + \dots + (nca_n - (n-1)a_{n-1})p_b^{n-1} - na_np_b^n \end{aligned} \quad (\text{A.3})$$

Grouping similar terms, and introducing a new coefficient  $a_{n+1} = 0$ , we get the following expression for first order condition, that is identical to Equation 3.17:

$$(n+1)(ca_{n+1} - a_n)p_b^n + n(ca_n - a_{n-1})p_b^{n-1} +$$

$$\dots + 2(ca_2 - a_1)p_b + (ca_1 - a_0) = 0 \quad (\text{A.4})$$

Similarly, for a seller, the objective function is given by Equation 3.19 and is as follows:

$$E_{lb}(p_s) = (p_s - c)(1 - a_0 - a_1p_s - a_2p_s^2 - \dots - a_{n-1}p_s^{n-1} - a_np_s^n) \quad (\text{A.5})$$

Applying first order conditions to the above equation and by introducing the coefficient  $a_{n+1} = 0$  in a manner similar to the approach for the buyer, we get the following result:

$$\begin{aligned} E'_{lb}(p_s) &= 0 \\ \Rightarrow (1 - a_0 - a_1p_s - a_2p_s^2 - \dots - a_{n-1}p_s^{n-1} - a_np_s^n) \\ &\quad + (p_s - c)(-a_1 - 2a_2p_s - \dots - (n-1)a_{n-1}p_s^{n-2} - na_np_s^{n-1}) = 0 \\ \Rightarrow 1 - a_0 - a_1p_s - \dots - a_{n-1}p_s^{n-1} - a_np_s^n \\ &\quad + ca_1 + (2ca_2 - a_1)p_s + \dots + (nca_n - (n-1)a_{n-1})p_s^{n-1} - na_np_s^n \quad (\text{A.6}) \end{aligned}$$

This leads to a similar condition for optimality as for the buyer, given by Equation 3.20, reproduced here:

$$\begin{aligned} (n+1)(ca_{n+1} - a_n)p_s^n + n(ca_n - a_{n-1})p_s^{n-1} + \\ \dots + 2(ca_2 - a_1)p_s + (ca_1 - a_0 + 1) = 0 \quad (\text{A.7}) \end{aligned}$$

### A.1.2 Incomplete-Beta Modeling

First order conditions for incomplete-beta modeling can be obtained by differentiating the objective functions for a buyer and a seller given by Equations 3.27, and 3.28 respectively, reproduced below:

$$E_{lb}(x) = (c - mx) \frac{1}{B(a, b)} \int_0^x t^{a-1} (1-t)^{b-1} dt \quad (\text{A.8})$$

For a seller, a very similar procedure would result in the following  $E_{lb}(x)$ :

$$E_{lb}(x) = (mx - c) \left( 1 - \frac{1}{B(a, b)} \int_0^x t^{a-1} (1-t)^{b-1} dt \right) \quad (\text{A.9})$$

First order conditions can be obtained by differentiating the above equations once w.r.t.  $x$ . The condition is as follows for a buyer:

$$E'_{lb}(x) = 0 \Rightarrow \frac{1}{B(a, b)} \left[ (c - mx)x^{a-1}(1-x)^{b-1} - m \int_0^x t^{a-1}(1-t)^{b-1} dt \right] = 0 \quad (\text{A.10})$$

Similarly for a seller:

$$E'_{lb}(x) = 0 \Rightarrow \frac{1}{B(a, b)} \left[ (c - mx)x^{a-1}(1-x)^{b-1} + m \left( 1 - \int_0^x t^{a-1}(1-t)^{b-1} dt \right) \right] = 0 \quad (\text{A.11})$$

### A.1.3 Incomplete-Gamma Modeling

First order conditions for incomplete-gamma modeling can be obtained by differentiating the objective functions for a buyer and a seller given by Equations 3.32, and 3.33 respectively, reproduced below:

$$E_{lb}(p_b) = (c - p_b) \frac{1}{\Gamma(a)} \int_0^{p_b} e^{-t} t^{a-1} dt \quad (\text{A.12})$$

$$E_{lb}(p_s) = (p_s - c) \left( 1 - \frac{1}{\Gamma(a)} \int_0^{p_s} e^{-t} t^{a-1} dt \right) \quad (\text{A.13})$$

First order conditions can be obtained by differentiating the above equations once w.r.t.  $p_b$  and  $p_s$  respectively. The condition is as follows for a buyer:

$$E'_{lb}(p_b) = 0 \Rightarrow \frac{1}{\Gamma(a)} \left[ (c - p_b)e^{-p_b} p_b^{a-1} - \int_0^{p_b} e^{-t} t^{a-1} dt \right] = 0 \quad (\text{A.14})$$

Similarly for a seller:

$$E'_{lb}(p_s) = 0 \Rightarrow \frac{1}{\Gamma(a)} \left[ (c - p_s)e^{-p_s} p_s^{a-1} + \left( 1 - \int_0^{p_s} e^{-t} t^{a-1} dt \right) \right] = 0 \quad (\text{A.15})$$

## A.2 Concavity Conditions for the Objective Function

### A.2.1 Polynomial Modeling

Concavity conditions for a maximum require that the second derivative of the objective function be negative. For polynomial modeling, this condition can be derived by differentiating Equations A.4 and A.7 w.r.t.  $p_b$  and  $p_s$  respectively, and the following results are obtained:

$$E''_{lb}(p_b) < 0 \Rightarrow (n+1)n(ca_{n+1} - a_n)p_b^{n-1} + n(n-1)(ca_n - a_{n-1})p_b^{n-2} + \dots + 2(ca_2 - a_1) < 0 \quad (\text{A.16})$$

$$E''_{lb}(p_s) < 0 \Rightarrow (n+1)n(ca_{n+1} - a_n)p_s^{n-1} + n(n-1)(ca_n - a_{n-1})p_s^{n-2} + \dots + 2(ca_2 - a_1) < 0 \quad (\text{A.17})$$

### A.2.2 Incomplete-Beta Modeling

Concavity conditions for incomplete-Beta Modeling are obtained for a buyer and a seller by differentiating Equations A.10 and A.11 w.r.t.  $x$  respectively. The result is the same for both cases, and is obtained as follows:

$$E''_{lb}(p_b) < 0 \Rightarrow \frac{1}{B(a, b)} \left[ -2mx^{a-1}(1-x)^{b-1} + (c-mx) \left\{ (a-1)x^{a-2}(1-x)^{b-1} - (b-1)x^{a-1}(1-x)^{b-2} \right\} \right] < 0 \quad (\text{A.18})$$

$$\Rightarrow \frac{x^{a-2}(1-x)^{b-2}}{B(a, b)} \left[ (a+b)mx^2 - \{(a+1)m + c(a+b-2)\}x + c(a-1) \right] < 0 \quad (\text{A.19})$$

In the above inequation, the first term on the left hand side,  $\frac{x^{a-2}(1-x)^{b-2}}{B(a, b)}$ , is always greater than zero for positive values of  $x, a, b$ . This condition is always satisfied

because  $x$  is a positive fraction of the maximum bid price  $m$ , and  $a$  and  $b$  are positive by definition for the beta distribution. Thus, the above condition can be simplified to yield the following quadratic inequation:

$$(a + b)mx^2 - \{(a + 1)m + c(a + b - 2)\}x + c(a - 1) < 0 \quad (\text{A.20})$$

This is identical to the concavity condition given by Equation 3.29.

### A.2.3 Incomplete-Gamma Modeling

Concavity conditions for incomplete-gamma modeling are obtained for a buyer and a seller by differentiating Equations A.14 and A.15 w.r.t.  $p_b$  and  $p_s$  respectively. The result is the same for both cases, and is obtained as follows, with  $p$  representing  $p_b$  for a buyer and  $p_s$  for a seller:

$$\begin{aligned} E''_{lb}(p) < 0 &\Rightarrow \\ \frac{1}{\Gamma(a)} \left[ -(c - p)e^{-p}p^{a-1} + e^{-p} \left\{ (a - 1)cp^{a-2} - ap^{a-1} \right\} - e^p p^{a-1} \right] &< 0 \\ \Rightarrow \frac{e^{-p}p^{a-2}}{\Gamma(a)} \left[ p^2 - (a + c + 1)p + (a - 1)c \right] &< 0 \end{aligned} \quad (\text{A.21})$$

In the above condition, the first term on the left hand side,  $\frac{e^{-p}p^{a-2}}{\Gamma(a)}$ , is always greater than zero for positive values of  $p$  and  $a$ . This condition is always satisfied because  $p$  is a positive bid price, and  $a$  is positive by definition for the gamma distribution. Thus, the above inequation can be simplified to yield the following quadratic inequation:

$$p^2 - (a + c + 1)p + c(a - 1) < 0 \quad (\text{A.22})$$

This is identical to the concavity condition given by Equation 3.34.

## BIBLIOGRAPHY

- [1] J. W. Lamont and S. Rajan, "Strategic bidding in an energy brokerage," Presented at the IEEE-PES Summer Meeting, 1997.
- [2] J. W. Lamont and S. Rajan, "Developing strategies for bidding in energy brokerages through simulation," in *Proceedings of the 59th Annual American Power Conference*, 1997.
- [3] A. K. David, "Competitive bidding in electric supply," *IEE Proceedings, Part C: Generation, Transmission and Distribution*, vol. 140, no. 5, pp. 421–426, 1993.
- [4] A. Saha, "Expo-power utility: A flexible form for absolute and relative risk aversion," *American Journal of Agricultural Economics*, vol. 75, pp. 905–913, November 1993.
- [5] J. Kumar and G. B. Sheblé, "A framework for transaction selection using decision analysis based upon risk and cost of insurance," in *Proceedings of the 26th Annual North American Power Symposium*, pp. 548–557, 1994.
- [6] M. Kriz, "Power brokers," *National Journal*, pp. 2594–2599, November 1996.
- [7] W. H. Dunn, M. A. Rossi, and B. Avramovic, "Impact of market restructuring on power systems operation," *IEEE Computer Applications in Power*, vol. 18, no. 1, pp. 42–47, 1995.



- [8] G. B. Sheblé. "Electric energy in a fully evolved marketplace," in *Proceedings of the 26th Annual North American Power Symposium*, pp. 81–90, 1994.
- [9] G. Stein, "Competitive sourcing of electricity," Presented at the Minnesota Power Systems Conference, 1994.
- [10] J. V. Barker, Jr., "Electric energy brokering: an explanation and status report," *Public Utilities Fortnightly*, pp. 28–36, February 4 1982.
- [11] K. W. Doty and P. L. McEntire, "An analysis of electric power brokerage systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, pp. 389–396, February 1982.
- [12] G. Fahd, D. A. Richards, and G. B. Sheblé, "The implementation of an energy brokerage system using linear programming," *IEEE Transactions on Power Apparatus and Systems*, vol. 7, pp. 90–96, February 1992.
- [13] G. Fahd and G. B. Sheblé, "Optimal power flow emulation of interchange brokerage system using linear programming," *IEEE Transactions on Power Apparatus and Systems*, vol. 7, pp. 497–504, May 1992.
- [14] D. Anwar and G. B. Sheblé, "Application of optimal power flow to interchange brokerage transactions," *Electric Power Systems Research*, vol. 30, pp. 83–90, 1994.
- [15] D. L. Post, S. S. Coppinger, and G. B. Sheblé, "Application of auctions as a pricing mechanism for the interchange of electric power," *IEEE Transactions on Power Systems*, vol. 10, no. 3, pp. 1580–1584, 1995.
- [16] D. Chattopadhyay, "An energy brokerage system with emission trading and allocation of cost savings," *IEEE Transactions on Power Systems*, vol. 10, pp. 1939–1945, November 1995.

- [17] S. R. Herriott, "The organizational economics of power brokers and centrally dispatched power pools," *Land Economics*, vol. 61, pp. 308–313, March 1985.
- [18] R. W. Hahn and V. Van Boening, "An experimental examination of spot markets for electricity," *The Economic Journal*, vol. 100, pp. 1073–1094, December 1990.
- [19] G. B. Sheblé and J. D. McCalley, "Discrete auction systems for power system management," Presented at the National Science Foundation Workshop, Pullman, WA, 1994.
- [20] K. A. McCabe, S. J. Rassenti, and V. L. Smith, "Smart computer-assisted markets," *Science*, vol. 254, pp. 534–538, 1991.
- [21] G. B. Sheblé, "Price based operation in an auction market structure," Presented at the IEEE-PES Winter Meeting, 1996.
- [22] J. Kumar and G. B. Sheblé, "Auction game in electric power market place," in *Proceedings of the 58th Annual American Power Conference*, vol. 58, pp. 1272–1277, 1996.
- [23] T. Alvey, D. Goodwin, X. Ma, D. Streiffert, and D. Sun, "A security-constrained bid-clearing system for the New Zealand wholesale electricity market," Presented at the IEEE-PES Summer Meeting, 1997.
- [24] H. Singh, S. Hao, and A. Papalexopoulos, "Power auctions and network constraints," in *Proceedings of the 30th Hawaiian International Conference on System Sciences*, pp. 608–614, 1997.
- [25] S. Jia and E. Radinskaia, "England and Wales power pool bidding strategy and the switching curve law," in *Proceedings of the Student Paper/Poster Session, 20th International Conference on Power Industry Computer Applications*, pp. 87–91, 1997.

- [26] S. S. Oren, A. J. Svoboda, and R. B. Johnson, "Volatility of unit commitment in competitive electricity markets," in *Proceedings of the 30th Hawaiian International Conference on System Sciences*, pp. 594–601, 1997.
- [27] S. Al-Agtash, "Resource operations planning in a PX market-based ISO restructuring mechanism," in *Proceedings of the Student Paper/Poster Session, 20th International Conference on Power Industry Computer Applications*, pp. 1–5, 1997.
- [28] J. Kumar and G. B. Sheblé, "Framework for energy brokerage system with reserve margin and transmission losses," Presented at the IEEE-PES Winter Meeting, 1996.
- [29] R. M. Stark and M. H. Rothkopf, "Competitive bidding: a comprehensive bibliography," *Operations Research*, vol. 27, no. 2, pp. 364–390, 1979.
- [30] L. Friedman, "A competitive-bidding strategy," *Operations Research*, vol. 4, pp. 104–112, 1956.
- [31] F. H. Griffis, "Bidding strategy: winning over key competitors," *Journal of Construction Engineering and Management*, vol. 118, no. 1, pp. 151–165, 1992.
- [32] D. P. Baron, "Incentive contracts and competitive bidding," *American Economic Review*, pp. 384–394, June 1972.
- [33] R. B. Wilson, "Competitive bidding with asymmetric information," *Management Science*, vol. 13, pp. 816–820, July 1967.
- [34] I. H. Lavalley, "A Bayesian approach to an individual player's choice of bid in competitive sealed auctions," *Management Science*, vol. 13, pp. 584–597, March 1967.
- [35] M. Skitmore and J. Pemberton, "A multivariate approach to construction contract bidding mark-up strategies," *Journal of the Operational Research Society*, vol. 45, no. 11, pp. 1263–1272, 1994.

- [36] P. D. Boughton, "The competitive bidding process: beyond probability models," *Industrial Marketing Management*, vol. 16, pp. 87–94, 1987.
- [37] R. P. Rozek, "Competitive bidding in electricity markets: a survey," *The Energy Journal*, vol. 10, no. 4, pp. 117–138, 1989.
- [38] E. P. Kahn, S. Soft, and D. Berman, "Optimization methods for evaluating competitive bids from private producers," *IEEE Transactions on Power Systems*, vol. 6, pp. 729–735, May 1991.
- [39] C. W. Richter, Jr., "Developing bidding strategies for electric utilities in a competitive environment," Master's thesis, Iowa State University, Ames, IA, 1996.
- [40] E. Sakk, R. J. Thomas, and R. Zimmerman, "Power system bidding tournaments for a deregulated environment," in *Proceedings of the 30th Hawaiian International Conference on System Sciences*, pp. 681–686, 1997.
- [41] S. Hao, G. A. Angelidis, H. Singh, and A. D. Papalexopoulos, "Consumer payment minimization in power pool auctions," in *Proceedings of the 20th International Conference on Power Industry Computer Applications*, pp. 368–373, 1997.
- [42] V. Krishna and V. C. Ramesh, "Intelligent agents for negotiations in market games, Part 1: model," in *Proceedings of the 20th International Conference on Power Industry Computer Applications*, pp. 388–393, 1997.
- [43] B. Prasannan, P. B. Luh, H. Yan, J. A. Palmberg, and L. Zhang, "Optimization based sale transactions and hydrothermal scheduling," in *Proceedings of IEEE Power Industry Computer Applications Conference*, pp. 137–142, 1995.
- [44] L. Zhang, P. B. Luh, X. Guan, and G. Merchel, "Optimization-based inter-utility power purchases," *IEEE Transactions on Power Systems*, vol. 9, pp. 891–897, May 1994.

- [45] J. Y. Fan, L. Zhang, and J. D. McDonald, "Enhanced techniques for sequential unit commitment with interchange transactions," *IEEE Transactions on Power Systems*, vol. 11, pp. 93–100, February 1996.
- [46] S. Takriti, J. R. Birge, and E. Long, "A stochastic model for the unit commitment problem," *IEEE Transactions on Power Systems*, vol. 11, pp. 1497–1508, August 1996.
- [47] A. M. Breiphol, F. N. Lee, D. Zhai, and R. Adapa, "A Gauss-Markov load model for application in risk evaluation and production simulation," *IEEE Transactions on Power Systems*, vol. 7, pp. 1493–1499, November 1992.
- [48] J. Hoffer and M. Prill, "On the models of peak load forecast uncertainty in probabilistic production costing algorithms," *International Journal of Electrical Power and Energy Systems*, vol. 18, no. 3, pp. 153–160, 1996.
- [49] R. Billinton and M. Fotuhi-Firuzabad, "Generating system operating health analysis considering stand-by units, interruptible load and postponable outages," *IEEE Transactions on Power Systems*, vol. 9, pp. 1618–1625, August 1994.
- [50] C. B. Lankford, J. D. McCalley, and N. K. Saini, "Bibliography on transmission access issues," *IEEE Transactions on Power Systems*, vol. 11, pp. 30–40, February 1996.
- [51] A. F. Vojdani, C. F. Imparato, N. K. Saini, B. F. Wollenberg, and H. H. Happ, "Transmission access issues," *IEEE Transaction on Power Systems*, vol. 11, pp. 41–51, February 1996.
- [52] S. T. Naumann, "The impact of FERC Orders 888 and 889 on transmission providers," in *Proceedings of the 59th Annual American Power Conference*, 1997.

- [53] H. Outhred and J. Kaye, "Incorporating network effects in a competitive electricity industry:- an Australian perspective," *The University of New South Wales School of Electrical Engineering, Department of Electric Power Engineering Report*, June 1994.
- [54] R. D. Tabors, "Transmission system management and pricing: new paradigms and international comparisons," Presented at the IEEE-PES Winter Meeting, 1993.
- [55] H. H. Happ, "Cost of wheeling methodologies," *IEEE Transactions on Power Systems*, vol. 9, pp. 147–156, February 1994.
- [56] Y. Tsukamoto and I. Iyoda, "Allocation of fixed cost to wheeling transactions by cooperative game theory," in *Proceedings of the Power Industry Computer Applications Conference*, pp. 3–10, May 1995.
- [57] A. Scarfone, "Short-circuit simulations help quantify wheeling flow," *IEEE Computer Applications in Power*, vol. 9, pp. 44–47, July 1996.
- [58] M. V. Obessis, J. D. McCalley, and J. W. Lamont, "On cost based transmission pricing," in *Proceedings of the 26th Annual North American Power Symposium*, pp. 116–125, 1994.
- [59] R. J. Thomas and T. R. Schneider, "Underlying technical issues in electricity deregulation," in *Proceedings of the 30th Hawaiian International Conference on System Sciences*, pp. 561–570, 1997.
- [60] R. E. Clayton and R. Mukerji, "System planning tools for the competitive market," *IEEE Computer Applications in Power*, vol. 9, pp. 50–55, July 1996.
- [61] N. Flatabo, "Software tools used in the Scandinavian market," Tutorial on Future Needs and Trends in Power System Computing, Presented at the 20th International Conference on Power Industry Computer Applications, 1997.

- [62] V. C. Ramesh and K. Ghosh, "Financial derivatives in power marketing," in *Proceedings of the 58th Annual American Power Conference*, pp. 912–917, 1996.
- [63] D. W. Carlton and J. M. Perloff, *Modern Industrial Organization*. New York: Harper Collins College Publishers, second ed., 1994.
- [64] V. Rothschild and N. Logothetis, *Probability Distributions*. New York: John Wiley & Sons, 1986.
- [65] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C, The Art of Scientific Computing*. Cambridge: Cambridge University Press, second ed., 1995.
- [66] J. M. Henderson and R. E. Quandt, *Microeconomic Theory – A Mathematical Approach*. New York: McGraw-Hill Book Company, third ed., 1980.
- [67] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*. New York: John Wiley & Sons, third ed., 1964.
- [68] J. W. Pratt, "Risk aversion in the small and in the large," *Econometrica*, vol. 32, pp. 122–136, January–April 1964.